PROBABILISTIC FUZZY CONTROLLER AS A GENERALIZATION OF THE CONCEPT OF FUZZY CONTROLLER

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Abstract

Two concepts of control algorithms are considered in this paper, i.e. a fuzzy controller and its generalization called probabilistic fuzzy controller.
Basic mathematical expressions for both of them are given pointing out that the concept of fuzzy controller is included in the concept of probabilistic fuzzy controller.
Original application of these control algorithms to biological processes is mentioned as well.

1. Introduction

A pioneering applicational paper concerning the fuzzy set theory in control was that of Mamdani [6]. This new approach to the theory and implementation of human control strategy was investigated by many researchers. Fuzzy sets were used to convert heuristic control rules stated by a human operator into automatic control algorithm. This algorithm called fuzzy controller is connected with two logical concepts, viz. fuzzy implication and compositional rule of inference. The rules arecomposed by means of fuzzy implications, where antecedents and consequences are treated as fuzzy sets.

Bearing in mind the existence of ambiguity and subjectivity factors dealing with a human operator, Czogała and Pedrycz [3] introduced the concept of probabilistic fuzzy controller, expressing the control strategy in terms of probabilistic sets proposed by Hirota [5].

Later on the idea of probabilistic fuzzy controller was generalized by Czogała [2].

Some aspects of synthesis of probabilistic fuzzy controller were presented by Czogała and Zimmermann [4].

The paper is set out as follows.

First of all basic expressions for fuzzy controller are provided.

Next basic expressions for probabilistic fuzzy controller are given.

After that a multi-dimensional case of a probabilistic fuzzy controller is considered.

Then some aspects of application in the control of biological processes are drafted.

Finally concluding remarks are formulated.

2. Basic expressions for fuzzy and probabilistic fuzzy controllers

There are various mathematical expressions for fuzzy controllers, i.e. from quite simple to very complex ones. It depends on the complexity of control statements (control rules) being taken into considerations. Assuming e.g. that the control statement has the form

$$\text{" if } A_i \text{ and } B_k \text{ then } C_{ik} \text{ "} \quad \text{for } i=1,\dots,p\ ;\ k=1,\dots,r \tag{2.1}$$

where the fuzzy sets A_i, B_k, and C_{ik} represent the error of the controlled variable, the change of the error and the output, respectively, the total control rule R, creating the memory of the controller, may be written as

$$R = \bigcup_{i,k} R_{ik} \tag{2.2}$$

where

$$R_{ik} = (A_i \times B_k) \times C_{ik} \tag{2.3}$$

The output of the controller C' is obtained by means of the compositional rule of inference, taking into account the inputs A' and B'

$$C' = B' \circ (A' \circ R) \tag{2.4}$$

or in terms of membership functions

$$C'(u) = \bigvee_{i,k} \bigvee_{y \in Y} \left[B'(y) \wedge \left(\bigvee_{x \in X} \left(A'(x) \wedge A_i(x) \wedge B_k(y) \wedge C_{ik}(u) \right) \right) \right] \tag{2.5}$$

Taking for example, crisp measurements as

$$A'(x) = \begin{cases} 1 & \text{if } x = x_0 \\ 0 & \text{otherwise} \end{cases}, \qquad B'(y) = \begin{cases} 1 & \text{if } y = y_0 \\ 0 & \text{otherwise} \end{cases}$$

we get a simplification in the form

$$C'(u) = \bigvee_{i,k} \left[A_i(x_0) \wedge B_k(y_0) \wedge C_{ik}(u) \right]$$

In the case when the inputs are denoted as $A' = A_j'$ and $B' = B_l'$ the output of the controller is obtained in the form

$$C_{jl}' = \bigcup_{i,k} \beta_{ijkl} \cap C_{ik} \tag{2.6}$$

In terms of the respective membership functions we get for the output the following expression

$$C'_{jl}(u) = \bigvee_{i,k} \beta_{ijkl} \wedge C_{ik}(u) \tag{2.7}$$

where the coefficients

$$\beta_{ijkl} = \Big(\bigvee_{x \in X} A_i(x) \wedge A'_j(x) \Big) \wedge \Big(\bigvee_{y \in Y} B_k(y) \wedge B'_l(y) \Big) \tag{2.8}$$

are called intersection coefficients [1] or degrees of separation [4]

In the relationship input-output of the fuzzy controller a structure with two levels is identified [1]. The first level is called fuzziness between the fuzzy sets of the controlled variable. The second level is called the fuzziness of the controlled statement

This has been illustrated in Fig. 1.

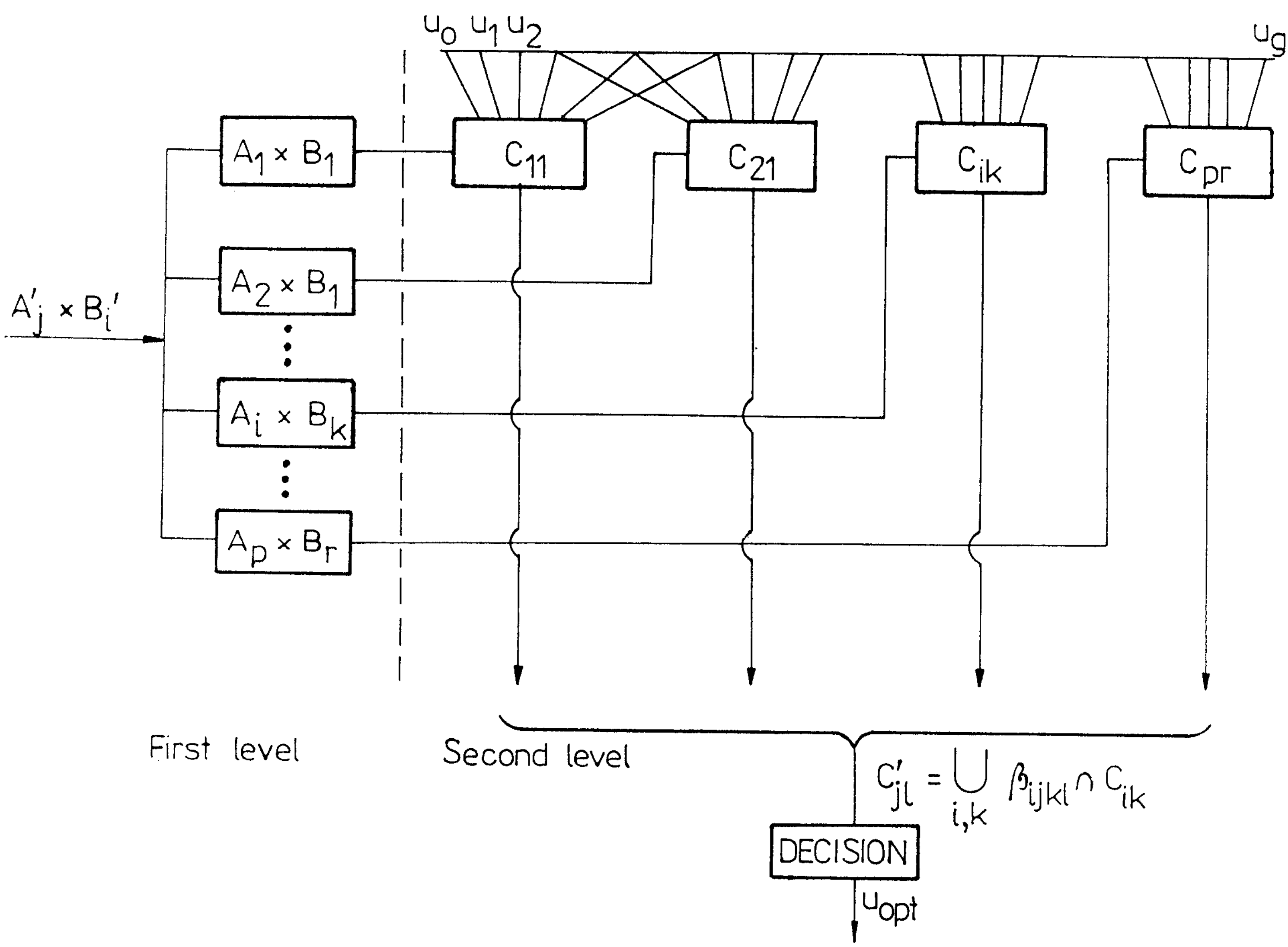

Fig.1.

A fuzzy controller will be called a probabilistic fuzzy controller, if at least one set represented in the aforesaid formulas is a probabilistic set.

Generally we can assume all sets are probabilistic sets defined by their defining functions i.e. $A_i(x,\omega)$, $B_k(y,\omega)$, $C_{ik}(u,\omega)$, $A'(x,\omega)$, $B'(y,\omega)$, $A'_j(x,\omega)$ and $B'_l(y,\omega)$

Taking into account the distribution function representation [2] of probabilistic sets and the operations on them we can write

$$F_{C'(u)}(z) = F_{\bigvee_{i,k} \bigvee_{y \in Y} \left[B'(y) \wedge \left(\bigvee_{x \in X} (A'(x) \wedge A_i(x) \wedge B_k(y) \wedge C_{ik}(u)) \right) \right]}(z) \tag{2.9}$$

or

$$F_{C'_{jl}(u)}(z) = F_{\bigvee_{i,k} \left(\beta_{ijkl} \wedge C_{ik}(u) \right)}(z) \tag{2.10}$$

An interesting case is obtained if we assume that the rules are independent and the sets A_i, A'_j, B_k, B'_l are fuzzy or crisp sets and C_{ik} are probabilistic sets. For this case we have

$$F_{C'_{jl}(u)}(z) = \prod_{i,k} \left[F_{\beta_{ijkl}}(z) \left(1 - F_{C_{ik}(u)}(z) \right) + F_{C_{ik}(u)}(z) \right] \quad \text{for } u \in U \text{ and } z \in [0,1] \tag{2.11}$$

We shall mention here that the formulas (2.9), (2.10), and (2.11) hold true for trivial random variables i.e. for the case when all sets are fuzzy only. In that case all probability distributions are stepwise functions.

We can observe two levels here as well. The first level is the fuzziness between the fuzzy subsets of the controlled variable as in the fuzzy controller and the second level is the probabilistic fuzziness of the control statement.

The formula (2.11) will be used in further considerations and practical applications.

Now we shall pass over to the deterministic value u_{opt} from the probability distributions $F_{C'_{jl}(u)}(z)$ for $u \in U$ and $z \in [0,1]$

We will take into account the stochastic dominance as in risky decision problems [7] . Suppose that two possible impacts of two alternatives u_m, $u_n \in U$ can be described by the probability distributions $F_{C'_{jl}(u_m)}(z)$ and $F_{C'_{jl}(u_n)}(z)$ on $[0,1]$, respectively.

Then the following axiom is approved under the expected utility criterion

$$u_m \succcurlyeq u_n \quad \text{means} \quad E\left[v,\ F_{C'_{jl}(u_m)}\right] \geqslant E\left[v,\ F_{C'_{jl}(u_n)}\right] \tag{2.12}$$

where the symbol $\succcurlyeq$ means " preferred or indifferent to ", and $E\left[v, (\cdot)\right]$ denotes mathematical expectation with respect to the utility function v and probability distribution $(\cdot)$ on $[0, 1]$ i.e.

$$E\left[v,\ F\right] = \int_0^1 v(z)\ dF(z) \tag{2.13}$$

Under the above axiom the probability distributions themselves are viewed as risky alternatives. As the available partial knowledge about the utility function the following classes of utility functions are defined

$$\mathcal{V}_1 = \left\{ v(z) \middle|\ v \in C^1\ ,\ \frac{dv}{dz}\ \ 0 \right\} \tag{2.14}$$

$$\mathcal{V}_2 = \left\{ v(z) \middle|\ v \in C^2\ ,\ v \in \mathcal{V}_1\ ,\ \frac{d^2v}{dz^2} \leqslant 0 \right\} \tag{2.15}$$

$$\mathcal{V}_3 = \left\{ v(z) \middle|\ v \in C^3\ ,\ v \in \mathcal{V}_2\ ,\ \frac{d^3v}{dz^3} \geqslant 0 \right\} \tag{2.16}$$

where C^i represents the set of bounded i-th differentiable functions. These classes are of importance for attitude of decisionmaker's preference toward risk. Obviously $\mathcal{V}_1$ is the class of utility functions for which the decisionmaker prefers an increase of the attribute level. $\mathcal{V}_2$ is the class for the decisionmaker to be risk-averse, and $\mathcal{V}_3$ is the class for the decisionmaker to be decreasingly risk-averse. With these classes the stochastic dominance is defined as follows.
For j=1,2 or 3, the distribution $F_{C'_{jl}(u_m)}$ dominates the distribution $F_{C'_{jl}(u_n)}$ in the sense of j-th degree stochastic dominance written as

$$F_{C'_{jl}(u_m)} \geqslant_j F_{C'_{jl}(u_n)} \quad \text{if} \quad E\left[v,\ F_{C'_{jl}(u_m)}\right] \geqslant E\left[v,\ F_{C'_{jl}(u_n)}\right] \quad \text{for} \ \forall v \in \mathcal{V}_j \tag{2.17}$$

The symbol $\geqslant_1$ refers to first-degree stochastic dominance, $\geqslant_2$ to second-degree stochastic dominance, and $\geqslant_3$ to third-degree stochastic dominance.

The necessary and sufficient conditions for stochastic dominance are the following ones [7].
Provided that $F_{C'_{j1}(u_m)}$ and $F_{C'_{j1}(u_n)}$ are distribution functions of a single variable,

1. $F_{C'_{j1}(u_m)} \geqslant_1 F_{C'_{j1}(u_n)}$ iff $F_{C'_{j1}(u_n)}(z) \geqslant F_{C'_{j1}(u_m)}$

$$\forall z \in [0,1] \qquad (2.18)$$

2. $F_{C'_{j1}(u_m)} \geqslant_2 F_{C'_{j1}(u_n)}$ iff $\int_0^z F_{C'_{j1}(u_n)}(t)\,dt \geqslant \int_0^z F_{C'_{j1}(u_m)}(t)\,dt$

$$\forall z \in [0,1] \qquad (2.19)$$

3. $F_{C'_{j1}(u_m)} \geqslant_3 F_{C'_{j1}(u_m)}$ iff $m_{F_{C'_{j1}(u_m)}} \geqslant m_{F_{C'_{j1}(u_n)}}$ and

$$\int_0^z \int_0^y F_{C'_{j1}(u_n)}(t)\,dt\,dy \geqslant \int_0^z \int_0^y F_{C'_{j1}(u_m)}(t)\,dt\,dy$$

$$\forall z \in [0,1] \qquad (2.20)$$

where $m_{(\cdot)}$ denotes the mean value of the distribution function $(\cdot)$ i.e.

$$m_F = \int_0^1 z\,dF(z)$$

3. Multi-dimensional probabilistic fuzzy controller

Instead of the multidimensional case of control statements in the form "if A_{1i} and B_{1k} and ... and A_{Mi} and B_{Mk} then $C_{.1ik}$ and ... and $C_{.Nik}$ "

$$(3.1)$$

where A_{1i} , B_{1k} ,...,A_{Mi} , B_{Mk} ; $C_{.1ik}$,...,$C_{.Nik}$ are probabilistic sets representing errors, their changes and outputs, repectively, we shall consider the interaction inside a probabilistic fuzzy controller as illustrated in Fig.2

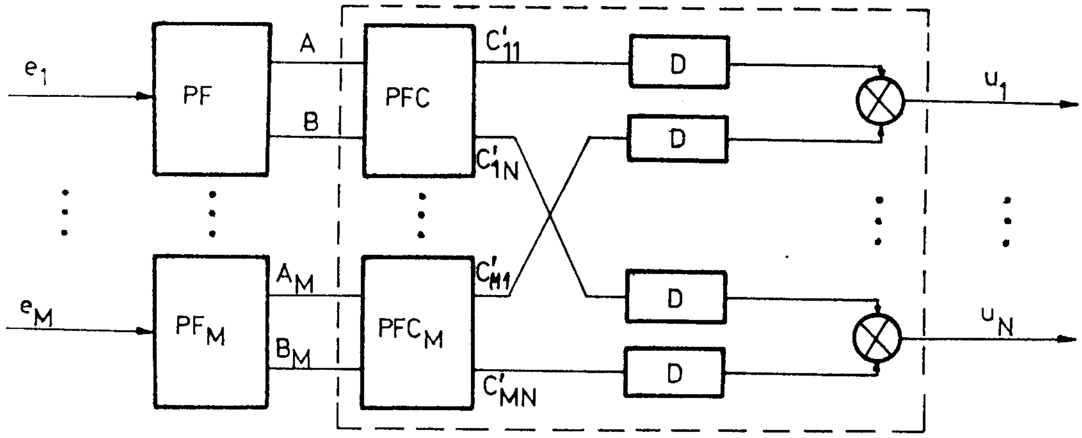

Fig.2.

It means that the partial outputs $C_{1qik},\dots,C_{Mqik}$ influence on the respective final output $C_{.qik}$ $q=1,\dots,N$, thus the meaning of the point in $C_{.qik}$ is clear enough, as well . Regardig this, it may be also assumed that the set of $p \cdot r$ rules of the form (3.1) will be replaced by the following collections of rules

$$
\begin{aligned}
&\text{" if } A_{1i} \text{ and } B_{1k} \text{ then } C_{11ik} \text{ ",}\\
&\quad\vdots\\
&\text{" if } A_{Mi} \text{ and } B_{Mk} \text{ then } C_{M1ik} \text{ ",}\\
&\quad\vdots\\
&\text{" if } A_{1i} \text{ and } B_{1k} \text{ then } C_{1Nik} \text{ ",}\\
&\quad\vdots\\
&\text{" if } A_{Mi} \text{ and } B_{Mk} \text{ then } C_{MNik} \text{ ".}
\end{aligned}
\tag{3.2}
$$

These collections form the basis for the decomposition proposed for fuzzy controllers and probabilistic fuzzy controllers, as well. Such a decomposition makes it possible to consider the controlling actions by the formulas obtained in the previous section i.e. in the case of a single input - single output controller. Also the inter actions are expressed by the similar formulas. Thus we may write as follows

$$
\begin{aligned}
C'_{11jl} &= \bigcup_{i,k} \beta^{1}_{ijkl} \cap C_{11ik} ,\\
C'_{M1jl} &= \bigcup_{i,k} \beta^{M}_{ijkl} \cap C_{M1ik} ,\\
C'_{1Njl} &= \bigcup_{i,k} \beta^{1}_{ijkl} \cap C_{1Nik} ,\\
C'_{MNjl} &= \bigcup_{i,k} \beta^{M}_{ijkl} \cap C_{MNik} .
\end{aligned}
\tag{3.3}
$$

where the first index denotes the input number and the second index denotes the output number of the controller.
For these formulas we can write analogous formulas in terms of the respective probability distriburions i.e.

$$F_{C'_{11jl}(u_1)}(z) = \prod_{i,k} \left[F_{\beta^1_{ijkl}}(z) \left(1 - F_{C_{11ik}(u_1)}(z)\right) + F_{C_{11ik}(u_1)}(z) \right],$$

$$F_{C'_{M1jl}(u_1)}(z) = \prod_{i,k} \left[F_{\beta^1_{ijkl}}(z) \left(1 - F_{C_{M1ik}(u_1)}(z)\right) + F_{C_{M1ik}(u_1)}(z) \right],$$

$$F_{C'_{1Njl}(u_N)}(z) = \prod_{i,k} \left[F_{\beta^1_{ijkl}}(z) \left(1 - F_{C_{1Nik}(u_N)}(z)\right) + F_{C_{1Nik}(u_N)}(z) \right],$$

$$F_{C'_{MNjl}(u_N)}(z) = \prod_{i,k} \left[F_{\beta^1_{ijkl}}(z) \left(1 - F_{C_{MNik}(u_N)}(z)\right) + F_{C_{MNik}(u_N)}(z) \right]. \tag{3.4}$$

As an example let us consider two-input and two-output control system illustrated in Fig.3 The final formulas for the probabilistic fuzzy controller of this system take the form

$$F_{C'_{11jl}(u_1)}(z) = \prod_{i,k} \left[F_{\beta^1_{ijkl}}(z) \left(1 - F_{C_{11ik}(u_1)}(z)\right) + F_{C_{11ik}(u_1)}(z) \right],$$

$$F_{C'_{21jl}(u_1)}(z) = \prod_{i,k} \left[F_{\beta^2_{ijkl}}(z) \left(1 - F_{C_{21ik}(u_1)}(z)\right) + F_{C_{21ik}(u_1)}(z) \right],$$

$$F_{C'_{12jl}(u_2)}(z) = \prod_{i,k} \left[F_{\beta^1_{ijkl}}(z) \left(1 - F_{C_{12ik}(u_2)}(z)\right) + F_{C_{12ik}(u_2)}(z) \right],$$

$$F_{C'_{22jl}(u_2)}(z) = \prod_{i,k} \left[F_{\beta^2_{ijkl}}(z) \left(1 - F_{C_{22ik}(u_2)}(z)\right) + F_{C_{22ik}(u_2)}(z) \right]. \tag{3.5}$$

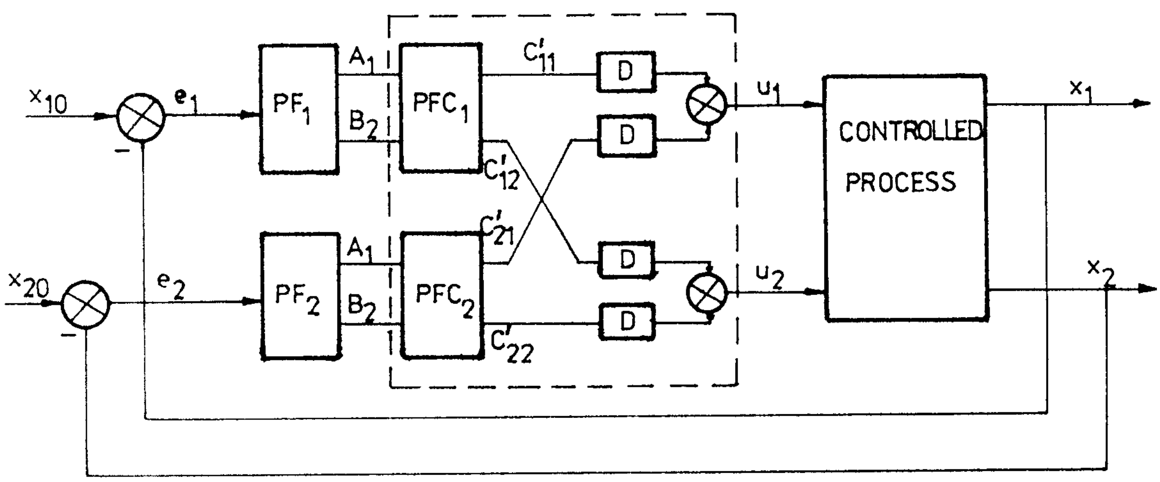

Fig.3.

4. Some applicational aspects of fuzzy and probabilistic fuzzy controllers in biological processes.

We will consider here a control task which may be treated by means of fuzzy and probabilistic fuzzy controllers i.e. a continuous cultivation of microorganisms used e.g. in bacteriological research, in the fermentation industry and in the biological treatment of municipal wastes. Such processes are complex enough and they are also ill-defined for several reasons. So the application of fuzzy and probabilistic fuzzy controllers seems to be appropriate because [8]

1° a complete understanding of the biological machanisms involved in biological processes is not available,

2° the measuring devices still cannot be used for on-line measurements for purposes of control, i.e. the existing sensors for biomass and substrate are not reliable and accurate enough.

Let us take the following closed loop feedback control shown in Fig. 4

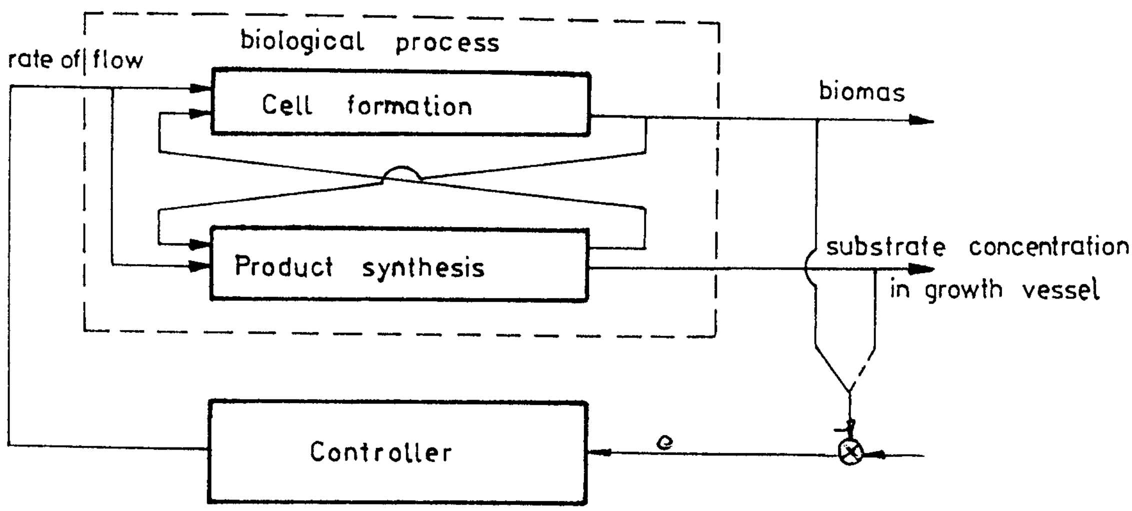

Fig. 4.

Taking into accont the poor quality of available measurements of the sensors (for biomass and the concentration of substrate in a growth vessel) a linguistical representation of these measurements may be used.

At present simulation results are obtained by means of Monod nonlinear equations [8] making use of the classical controllers PI, PID and fuzzy or probabilistic fuzzy controllers

The results are still better when classical controllers are used. But this is the case when we have exact measurements (the error and the change of error are calculated by means of differential equations using for their solution Runge-Kutta's method of the forth order). In the case when the measurements of outputs of the process are of poor quality the situation is different. As classical control methods require well defined concepts, precision and exact data, they cannot be used in this case. Moreover fuzzy and probabilistic fuzzy controllers can be also used when exact measurements at all not available (they are only estimated by a human operator). In such a case the control of the process with a human operator in the feedback loop must be taken into accont.

5. Concluding remarks

The paper presents the basic expressions for the fuzzy and probabilistic fuzzy controllers. These expressions show that the concept of fuzzy controller is embedded in the concept of probabilistic fuzzy controller.

The formulas for outputs of the probabilistic fuzzy controller presented here are uniform for one-dimensional and multi-dimensional cases.

The expressions for probabilistic fuzzy controller given above assure effective computability as well.

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