DEFAULT PRODUCTION RULES

Witold PEDRYCZ
Dept.of Automatic Control and Computer Sci.
Silesian Technical University,Gliwice
and
Research and Development Center of Electronic
Medical Instr.,Zabrze,Poland

Abstract The paper deals with"if-then"default production rules containing fuzzy premises.Their default values are expressed in a numerical fashion by testing a consistency of the rules against the knowledge base formed by means of respective system of fuzzy relational equations.

Keywords:default reasoning,production rules,fuzzy relational equation

1. Introduction

In this paper we focus an attention to analysis of production rules"if-then" type with fuzzy antecedents and consequents.We discuss one of possible ways of algorithmic assignment of default value to every production rule.This allows to order the rules with respect to their range of applicability. There is no doubt that default reasoning(cf. e.g. [5],also studied in setting of fuzzy sets, [4])as well as expert systems making use of the production rules play a significant role in knowledge engineering and requires further extensive investigations.

It is well known fact that the commonly discussed production rules with fuzzy data are of the form,

$$\text{if } \underline{X}_i \text{ then } Y_i, i=1,2,\dots,N \qquad /1/$$

where $\underline{X}_i$ and Y_i are treated as fuzzy relations and fuzzy sets, respectively.More precisely,

$\underline{X}_i: \bigtimes_{k=1}^{K} \mathbb{X}_k \rightarrow [0,1] \quad , Y_i: \mathbb{Y} \rightarrow [0,1] \quad .$ /2/

Usually to every production rule a certain measure of certainty is attached. For instance in MYCIN [7], we deal with a certainty factor /CF/ ascribed to the production rule. In [4] instead of the production rules /1/, default production rules are discussed,

usually(if $\underline{X}_i$ then Y_i) . /3/

A main question concerns the form of quantitative representation of the fuzzy term "usually" /or similar ones: seldom, often etc./ Using some results that were achieved while discussing a problem of solvability of a system of fuzzy relational [6] equations /for some previous results see for example [1]-[3] / we are able to provide the reader with a numerical representation of the default values of the production rules.

2. Solvability of the system of fuzzy relational equations and default production rules

Formally speaking the set of the production rules specified above are put down,

$\underline{X}_i \circ R = Y_i \quad , i=1,2,\ldots,N$ /4/

where a fuzzy relation $R: \bigtimes_{j=1}^{K} \mathbb{X}_j \times \mathbb{Y} \rightarrow [0,1]$ combines all of them. Further on, to avoid some secondary details in a main stream of discussions we restrict ourselves to the rules

$X_i \rightarrow Y_i$ viz. $X_i \circ R = Y_i$ with $X_i: \mathbb{X} \rightarrow [0,1]$, $i=1,2,\ldots,N$.

Let us prove the following useful lemma.

Lemma. If

$$\forall_{y \in \mathbb{Y}} \quad \exists_{x \in \mathbb{X}} \quad X(x) \geqslant Y(y) \qquad /5/$$

then the equation $X \circ R = Y$ has a solution, namely, $\mathcal{R} = \{R: \mathbb{X} \times \mathbb{Y} \rightarrow [0,1] \mid X \circ R = Y\} \neq \emptyset$.

Proof. We put down the fuzzy relation $\hat{R} = X \circledast Y$ and prove that under the assumption /5/, $\mathcal{R}$ is nonempty. By definition of sup-min composition "$\circ$", and α-composition ($a \alpha b = 1$, if $a \leqslant b$, and b, if $a > b$),

$$\sup_x [X(x) \wedge \hat{R}(x,y)] = \sup_x [X(x) \wedge (X(x) \alpha Y(y))] = \max\{ \sup_{x: X(x) > Y(y)} [X(x) \wedge (X(x) \alpha Y(y))] ,$$

$$\sup_{x: X(x) \leqslant Y(y)} [X(x) \wedge (X(x) \alpha Y(y))] \} = \max\{ \sup_{x: X(x) > Y(y)} Y(y) , \sup_{x: X(x) \leqslant Y(y)} X(x) \} = Y(y) \qquad /6/$$

In sequel a proposition below specifies condition to achieve solvability of the system of equations $X_i \circ R = Y_i$, $i=1,2,\ldots,N$.

Proposition

If

i/ X_i, Y_i satisfy the equation $X_i \circ R = Y_i$ for all $i=1,2,\ldots,N$.

ii/ X_i are pairwise disjoint

$$\underset{i \neq j}{\forall} \quad X_i \cap X_j = \emptyset \qquad /7/$$

then the system of the fuzzy relational equation given above has a solution, $\mathcal{R} = \{R \mid X_i \circ R = Y_i \ , i=1,2,\ldots,N\} \neq \emptyset$.

Proof. In virtue of ii/, we get $\mathrm{supp}(X_i) \cap \mathrm{supp}(X_j) = \emptyset$ for all $i \neq j$ $(\mathrm{supp}(X_i) = \{x \mid X_i(x) > 0\})$. We prove that

$$\hat{R} = \bigcap_{i=1}^{N} \hat{R}_i \quad , \hat{R}_i = X_i \, \textcircled{\alpha} \, Y_i \qquad /8/$$

forms the element of $\mathcal{R}$.

Thus,

$$\hat{R}(x,y) = \min_j [X_j(x) \alpha \ Y_j(y)] \qquad /9/$$

and

$$\hat{R}(x,y) = \hat{R}'(x,y) \wedge \hat{R}_i(x,y) \qquad /10/$$

where

$$\hat{R}'(x,y) = \bigwedge_{j \neq i} (X_j(x) \ \alpha \ Y_j(y)) \qquad /11/$$

For all $x \notin \mathrm{supp}(X_i)$ we get $\hat{R}'(x,y) = 1.0$ for $y \in \mathbf{Y}$. Thus

$$\sup_x [X_i(x) \wedge \hat{R}(x,y)] = \sup_x [X_i(x) \wedge \hat{R}_i(x,y) \wedge \hat{R}'(x,y)] =$$

$$= \max\{ \sup_{x \in \mathrm{supp}(X_i)} [X_i(x) \wedge \hat{R}_i(x,y) \wedge \hat{R}'(x,y)] \ , \sup_{x \notin \mathrm{supp}\, X_i} [X_i(x) \wedge \hat{R}_i(x,y) \wedge \hat{R}'(x,y)] \} =$$

$$= \sup_{x \in \mathrm{supp}(X_i)} [X_i(x) \wedge \hat{R}_i(x,y)] \qquad /12/$$

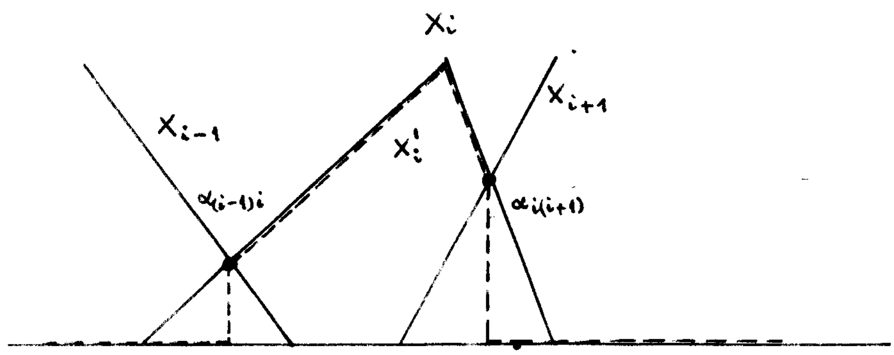
which bearing in mind i/, yields $Y_i(y)$, $y \in \mathbf{Y}$.

This, in turn, suggests the following modification in the production rules which concerns replacing in the production rules the fuzzy sets of antecedents by disjoint fuzzy sets. Instead of the original fuzzy sets $X_1, X_2, \ldots, X_n$ take their modifications $X_1', X_2', \ldots, X_n'$ which lead to $X_i' \rightarrow Y_i$, $i=1,2,\ldots,N$. Now the previous rules are default ones, should be read as

$$\lambda_i \ (X_i \rightarrow Y_i) \qquad /13/$$

(e.g. usually (if X_i then Y_i)). In such a sense $X_i' \rightarrow Y_i$ are interpreted as free from the default values, or viewed as: in every case (if X_i' then Y_i). The values of the linguistic notion for the default values can be determined with the aid of the procedure that "disconnets" X_i's.

Let X_i are unimodal fuzzy sets in $\mathbb{R}$ (which is rather a mild assumption). The corresponding fuzzy sets X_i' are constructed according to the following formula, see also a figure below .



$$X_i'(x) = \begin{cases} X_i(x) \text{ if } X_i(x) \geq \alpha_{i-1} \text{ and } x \leq x_i \\ X_i(x) \text{ if } X_i(x) \geq {}_{i+1} \text{ and } x \geq x_i \\ 0, \text{otherwise} \end{cases} \qquad /14/$$

and $X_i(x_i) = \sup_x X_i(x)$, $\alpha_{i-1} = \max_{j: x_j < x_i} \alpha_{ij}$ $\quad \alpha_{i+1} = \max_{j: x_j > x_i} \alpha_{ij}$

$\alpha_0 = \alpha_{N+1} = 0$, $\alpha_{ij} = \text{Poss}(X_i | X_j) = \sup_x (X_i(x) \wedge X_j(x))$

Notice that such a construction ensures us to achieve the whole universe of discourse is "covered" by X_i' 's , $\max_i X_i'(x) > 0$ for all x (if, of course, the family of X_i''s does it). Then to every rule $X_i \rightarrow Y_i$ we can assign an interval $[\alpha_{i-1} \wedge \alpha_{i+1}, \alpha_{i-1} \vee \alpha_{i+1}]$ which creates a numerical representation of the default values specified for the production rules. In this situation,

$$X_i \xrightarrow{\xi} Y_i \quad \text{with} \quad \xi = [\alpha_{i-1} \wedge \alpha_{i+1}, \alpha_{i-1} \vee \alpha_{i+1}]$$

Of course, if $\alpha_{i-1} = \alpha_{i+1} = 0$ this implies the rule : in every case (if X_i then Y_i) . In the worst case, $\alpha_{i-1} = \alpha_{i+1} = 1$ one has to discard the rule.

For the antecedents expressed in the cartesian product $\underline{X}_i \rightarrow Y_i$, this procedure may be generalized accordingly: repeat the above algorithm /14/ for every coordinate of $\underline{X}_i$'s /$\underline{X}_i = X_{i1} \times X_{i2} \times \ldots \times X_{iK}$/ obtaining α_{ik}, i=0,1,...,N,N+1, k=1,2, ...,K. Combine them together putting

$$\beta_{i-1} = \bigwedge_{k=1}^{K} \alpha_{/i-1/k} \wedge \bigwedge_{k=1}^{K} \alpha_{/i+1/k} \qquad /15/$$

$$\beta_{i+1} = \bigvee_{k=1}^{K} \alpha_{/i-1/k} \vee \bigvee_{k=1}^{K} \alpha_{/i+1/k} \qquad /16/$$

Thus,

$$\underline{X}_i \xrightarrow{[\beta_{i-1},\ \beta_{i+1}]} Y_i \qquad /17/$$

References

1. S.Gottwald,Generalized solvability criteria for systems of fuzzy equations defined with t-norms,BUSEFAL,no.21,1985,77-81.
2. S.Gottwald,W.Pedrycz,Solvability of fuzzy relational equations and manipulation of fuzzy data,Fuzzy Sets and Systems,to appear
3. W.Pedrycz,Applications of fuzzy relational equations for methods of reasoning in presence of fuzzy data,Fuzzy Sets and Systems,16,1985,163-175.
4. H.Prade,Reasoning with fuzzy default values. Proc. 15th Inter. Symp. Multiple-Valued Logic ,Kingston ,On,pp. 191-197,May 1985
5. R.Reiter,On reasoning by default,Proc.Second Symp.on Theoretical Issues in Natural Language Processing,Urbana,Illinois, July 25-27,1978.
6. E.Sanchez,Resolution of composite fuzzy relation equations. Information and Control,30,1976,38-48.
7. E.H.Shortliffe,Computer-Based Medical Consultations:MYCIN. Elsevier,New York,1976.