ON SOME PELATION BETWEEN FUZZY PROBABILITY MEASURE AND FUZZY P-MEASURE

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Let $\sigma = \{\mu : \Omega \to [0,1]\}$ be a soft fuzzy σ-algebra i.e. fuzzy σ-algebra (see [1]) uncontaining the fuzzy subset $[\![\frac{1}{2}]\!]_\Omega : \Omega \to \{\frac{1}{2}\}$.
Let us compare the following notions:

Definition 1: A fuzzy probability measure is a mapping

$$m: \sigma \to [0,1]$$

such that

$$m(\mathbb{0}_\Omega) = 0 \quad ,$$

$$m(\mathbb{1}_\Omega) = 1 \quad ,$$

$$\forall_{(\mu,\nu) \in \sigma^2} \qquad m(\mu \vee \nu) + m(\mu \wedge \nu) = m(\mu) + m(\nu) \quad ,$$

$$\forall_{\{\mu_n\} \in \sigma^N} \qquad \{\mu_n\} \uparrow \mu \Rightarrow \{m(\mu_n)\} \uparrow m(\mu) \quad .$$ [2]

Definition 2: Each mapping

$$p: \sigma \to R^+ \cup \{0\}$$

having the following properties:

- for any $\mu \in \sigma$

$$p(\mu \vee (1-\mu)) = 1$$

- if $\{\mu_n\}$ is finite or infinite sequence of pairwise W-separated fuzzy subsets from σ (i.e. $\mu_i \leq 1 - \mu_j$ for each pair (i,j) which $i \neq j$)

then

$$p\left(\sup_n\{\mu_n\}\right) = \sum_n p(\mu_n)$$

is called a fuzzy P-measure. [6]

As we know, the fuzzy P-measure are the unique fuzzy probability measures satisfying the Bayes Formula [6,7]. The next relationship between measures mentioned above, will be presented in this paper.

Let $\bar{\mathbb{R}} = [-\infty, +\infty]$. Then we have.

Definition 3: A fuzzy relation "less or equal" FLE is a mapping

$$\varrho : \bar{\mathbb{R}}^2 \to [0,1]$$

such that

$$\varrho(x,y) \geqslant 1 - \varrho(y,x) \quad ,$$

$$\varrho(y,x) + \varrho(z,y) \leqslant 1 \quad ,$$

for each $(x,y,z) \in \bar{\mathbb{R}}^3$ which $x < z$. [3]

Any FLE ϱ generates a fuzzy relation "less than" given by the identity

$$\varrho_s(x,y) = 1 - \varrho(y,x)$$

for every $(x,y) \in \bar{\mathbb{R}}^2$ [3].

Let ϱ be a fixed FLE.

Definition 4: Each mapping:

$$\varphi\langle a,b\rangle : \bar{\mathbb{R}} \to [0,1]$$

defined by the identity

$$\varphi\langle a,b\rangle(x) = \psi(a,x) \wedge \eta(x,b)$$

for every $(a,b,x) \in \bar{\mathbb{R}}^3$, $(\psi, \eta) \in \{\varrho, \varrho_s\}^2$, is called a fuzzy interval. [4]

We note, that the above definition describes all kinds of intervals on real line $\mathbb{R}$ generalized for fuzzy case. Among other things, it defines

- if $\psi = \varrho$ and $\eta = \varrho$ then $\varphi\langle a,b\rangle = \varphi[a,b]$;
- if $\psi = \varrho$ and $\eta = \varrho_s$ then $\varphi\langle a,b\rangle = \varphi[a,b[$.

Let us suppose now, that FIE ϱ is quasi-antisymmetrical, continuous from above and it unfuzzily bounds the real line (see [3]). Then there exists the smallest soft fuzzy σ-algebra containing all fuzzy intervals $\varphi[-\infty,a[$, $\mathcal{B}_\varrho$ say [4]. If $m: \mathcal{B}_\varrho \to [0,1]$ is a fixed fuzzy probability measure on $\mathcal{B}_\varrho$, then we define:

Definition 5: The cumulative distributions function of fuzzy probability measure m is a mapping

$$F: \mathbb{R} \to [0,1]$$

defined as

$$\forall x \in \mathbb{R} \qquad F(x) = m(\varphi[-\infty,x[) \ . \qquad [5]$$

Theorem 1: Each cumulative distribution function F fulfils the following conditions:

$$\forall (x,y) \in \mathbb{R}^2 \qquad x \leq y \Rightarrow F(x) \leq F(y) \ , \tag{1}$$

$$\forall \{x_n\} \in \mathbb{R}^N \qquad \{x_n\} \uparrow x \Rightarrow \{F(x_n)\} \uparrow F(x) \ , \tag{2}$$

$$\lim_{x \uparrow +\infty} F(x) = 1 = F(+\infty) \ , \tag{3}$$

$$\lim_{x \uparrow -\infty} F(x) = \alpha \geq 0 = F(-\infty) \ . \quad [5] \tag{4}$$

Theorem 2: If a function $f: \mathbb{R} \to [0,1]$ fulfils the properties (1), (2), (3) and (4) then there exists the unique fuzzy P-measure $p: \mathcal{B}_\varrho \to [0,1]$ such that

$$\forall x \in \mathbb{R} \qquad p(\varphi[-\infty, x[) = f(x) \ ,$$
$$p(\varphi[+\infty, +\infty]) = 0 \quad [8] \ . \qquad (5)$$

More details about fuzzy P-measure p generated by function f we can find in [8] . If we take into account two above theorems, then we obtain finally thesis.

Theorem 3: For each fuzzy probability measure $m: \beta_\varsigma \to [0,1]$, there exists the unique fuzzy P-measure $p: \beta_\varsigma \to [0,1]$ satisfing the conditions (5) and

$$\forall x \in \mathbb{R} \qquad p(\varphi[-\infty, x[) = m(\varphi[-\infty, x[) \ .$$

By means of this theorem, the Bayes Formula can be applied for any fuzzy probability space defined by Klement et.al. [2] .

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