PANSYSTEMS METHODOLOGY AND NONLINEAR ANALYSIS: NEW STUDIES OF BIFURCATION, CATASTROPHE, CHAOS AND STABILITY

Wu Xuemou
Wuhan Digital Engineering Institute, China

Pansystems methodology (PM) is also called pansystems theory or pansystems research, it is a kind of transfield investigation and applications of things mechanism or generalized systems with the emphasis of relations, relation transformations and pansymmetry, in particular, its key points are the mathematical studies on the so-called pansystems relations (or PM relations) and their intertransformations, including twelve important, universal and fundamental things relations and their simulations to various things mechanisms. Typical PM relations are concerned, for example, with macromicroscopy, motion-rest, whole-parts, body-shadow, causality, observocontrol, shengke (synergy-conflict), panorder, series-parallel, simulation, clustering-discoupling, difference-identity, etc. Pansystems methodology embodies partially certain new interpermeation, unification and generalization of mathematics-physics science, systems science, cognition science and social science, including certain combinations of Chinese traditional philosophy theory and modern sciences, and possesses reinterdisciplinary and retransdisciplinary nature for many metadisciplines, interdisciplines and transdisciplines. PM was presented first in 1976 in China, and now more than 10 research groups were established. The related contents investegated include more than 30 topics and many new concrete results are obtained. PM gets many hundreds of new theorems or mathematical propositions, and complements or develops many tens of affirmed researches, including some fundamental results in analysis mathematics, topology, discrete mathematics, mathematical logic, operations research, cybernetics, universal algebra and group theory, etc., so we can from some new viewpoints extend studies on nonlinear problems which connect closely with some new exploration of bifurcation, catastrophe, chaos and stability.

The mathematics in PM is like discrete one which does not need specialized analytic, topological and linear structures, the related functions, relations, operators, equations and even if fundamental universe or basic space all are nonlinear, consequently, PM is rather distinct from traditional nonlinear analysis.

PM establishes many mathematical models for pansystems relations and investigates their properties and transforming relations, their substance is nonlinear. The transformations which reform things, knowledge, information and structures into PM relations all are called PM operators. By using PM operators we can complement, extend or develop many important studies concerned, for example, with group representation, algorithm of maximum network-flow, clustering, division into district, hierarchy principle, black-box principle, grey-box principle, discoupling principle, implicit function theorem, fixed point theorem, homomorphism theorem, Dilworth-type theorem, cognitive processes, data-pressing, Arrow's impossibility theorem, Debreu's price equilibrium theorem, observocontrollability, independence-completeness-consistence of axiomatic systems, ecological law, phenological law, large scale systems, bifurcation, catastrophe, chaos and stability, etc. In what follows we only concern with the PM operator analysis which is closely connected with this paper.

Let $E_s[G)$, $E[G)$ be the classes of tolerance and equivalence relations on the given set G respectively. If $\delta \in E_s[G)$, define $G_i = \max\{D \mid D \subset G, D^2 \subset \delta\}$, and denote $G = \cup G_i(d\delta)$, $G_i \subset G(d\delta)$, $f_\delta = \{(x, G_i) \mid x \in G_i\}$, $G/\delta = \{G_i\}$. Clearly, $f_\delta \subset G \times (G/\delta)$, and we have

Theorem 1. $G = \cup G_i$, and $(x,y) \in \delta$ equivalent to $\exists i (x, y \in G_i)$. $f_\delta \circ f_\delta^{-1} = \delta$, $f_\delta^{-1} \circ f_\delta = \{(G_i, G_j) \mid G_i \cap G_j \neq \phi\} \in E_5[G/\delta)$. $\delta \in E[G)$ makes $\cup G_i$ distinct, and $f_\delta : G \rightarrow f_\delta(G) = G/\delta$, $(x,y) \in \bar{\delta}$ to be equivalent to $\exists ij (i \neq j, x \in G_i, y \in G_j)$, and $x \circ \delta = \delta \circ x = y \circ \delta = \delta \circ y = G_i$.

Theorem 2. E_5-class is closed or conservative for conjunction, disjunction, inverse relation, commutative composition, confinement, eqimorphism, epiembodiment, epibinary relation, transitive closure. And so E -class for conjunction, inverse relation, commutative composition, confinement, epiembodiment and transitive closure.

Let $g \in P(G^2)$, define PM operators $\varepsilon_1(g) = g \cup g^{-1} \cup I$, $\varepsilon_2(g) = \varepsilon_1(g \cap g^{-1})$, $\varepsilon_3(g) = \varepsilon_1(g^t \cap g^{-t})$, $\varepsilon_{i+5}(g) = \varepsilon_i(G^2 - g)$, $\delta_j(g) = [\varepsilon_j(g)]^t$, $i, j = 1, 2, 3$, where $I = I(G) = \{(x,x) \mid x \in G\}$, and index t means transitive closure. Let $f \subset G \times H$, we define its panratio as $f' : P(G^2) \rightarrow P(H^2)$ with the details: $f'(g) = f^{-1} \circ g \circ f$. In what follows, we use $L[G)$, $L_c[G)$, $L_5[G)$, $U[G)$ to represent the classes of semiordered, complete ordered, reflexive-transitive and unidirectional relations on G respectively. We have

Theorem 3. $\varepsilon_i(g) \in E_5[G)$, $\delta_j(g) \in E[G)$, $f_\delta'(g) \subset I(G/\delta)$ for $\delta = \delta_1(g)$, $f_\delta'(g) \in U[G/\delta)$ for $\delta = \delta_3(g)$. If $g \in L_5[G)$, $\delta = \varepsilon_2(g)$, then $f_\delta'(g) \in L[G/\delta)$. If $g \in L[G)$, $G = \cup G_k(d\varepsilon_1(g)) = \cup G_m'(d\varepsilon_6(g))$, then G_k are of maximal chains of g, $g \cap G_k^2 \in L_c[G_k)$, and G_m' are of maximal antichains of g.

Consequently, $\delta_1(g)$ realizes a black-box for g and $\delta_3(g)$ makes g a unidirectional grey-box. These concepts mentioned are simply called δ (1,3)-discoupling principle. Furthermore, $\varepsilon_6(g)$ makes some properties to be changed to corresponding opposites for difference-identity and clustering-discoupling relations (CD-transforming principle). Usually, practical problems are described by panweigted relations which can be reduced to some binary relations, and so to E_5 or E classes by using PM operators defined above. Based on these reductions, we can establish many intertransformations among some concepts concerning pansystems relations. This law described is called

PM correspondence principle. By using these PM principles, we can obtain a class of mathematical propositions (called BC^2S -theorems) concerning bifurcation, catastrophe, chaos, stability, uniqueness, single valuedness, multivaluedness, qualitative or quantitive change, sudden or gradual change.

Let $T \subset \prod T_\sigma$ be parameter set, $F = \{ x(t) | t \in T \}$ be given system set or structure set, $\varphi : F \to P(G^2)$, $\theta = \theta(t) = \varphi(x(t)) \subset G^2$, $\theta^{(n)}$ be the n-th self-composition of θ, $D \subset G(d\delta)$, $\delta \in E_5[G)$, $R[D)$: reflexive relation class on D. We can prove following BC^2S-theorem.

Theorem 4. If D does not be reduced to a single point, $\mathcal{E}_1(\theta) \subset \delta$, then $D \cap (D \circ \theta \cup \theta \circ D) \neq \phi$. If $\mathcal{E}_2(\theta) \subset \delta$, then $D \cap D \circ \theta \cap \theta \circ D \neq \phi$. If $\mathcal{E}_3(\theta) \subset \delta$, then there exist positive integers m, n, such that $D \cap D \circ \theta^{(m)} \cap \theta^{(n)} \circ D \neq \phi$. If $\delta \subset \mathcal{E}_6(\theta)$, $\theta \cap I = \phi$, then $D \cap (D \circ \theta \cup \theta \circ D) = \phi$. If $\delta_1(\theta) \subset \delta$, then $D \circ \theta \cup \theta \circ D \subset D$. If $\theta \subset \delta$, $\theta \circ \theta^{-1} \in R[D)$, then $\theta \circ D = D$. If $\delta_4(\theta) \subset \delta$, $\theta^{-1} \circ \theta \in R[D)$, then $D \circ \theta = D$.

Let $\rho = (F, G, Q, \psi, \eta, \xi)$, $\xi = (\varphi, \theta, t, D, \delta, F, G)$, $\psi \subset Q^2$, $\eta : Q \to \{\xi\}$, then ρ describes some multiparameter nature of various behaviors of BC^2S.

If $g \subset G^2$, $D \subset G$, $D^2 \cap g = \phi$, then D is called panchaos or interior stability for g. If for all $x \in G - D$, $x \circ g \cap D \neq \phi$, then D is called panattractor or exterior stability for g. If D is both panchaos and panattractor for g, then D is called strange panattractor or core for g. These definitions lead to following BC^2S-theorems.

Theorem 5. If D is a panchaos for g, then $D^2 \subset \mathcal{E}_7(g)$, $D \subset (G - D \circ g) \cap (G - g \circ D)$, $D \cap (D \circ g \cup g \circ D) = \phi$, $D \circ g \cup g \circ D \subset G - D$, $D \subset \mathcal{E}_7(g) \circ D \cap D \circ \mathcal{E}_7(g)$.

Theorem 6. If D is a strange panattractor, then $D \subset G(d\mathcal{E}_7(g))$ and vice versa for g being symmetry and $g \cap I(G) = \phi$.

Theorem 7. If G is finite, $g \circ (g^{(2)})^t \cap I(G) = \phi$, then there exists a strange panattractor D for g, $D \subset G(d\mathcal{E}_7(g))$.

Theorem 8. If $C(x)$ is the characteristic function of D, and $C(x) = 1 - max\{C(y) | y \in x \circ g\}$, then D is a strange panattractor for g , and $D\subset G(d\mathcal{E}_7(g))$. The inverse proposition holds for $g=g^{-1}$, $g\cap I(G)=\phi$.

Theorem 9. If $D\subset G(d\delta)$, $g\subset\delta$, $\delta\in E[G)$, and D is a panchaos for g^t, then D is also a strange attractor for g , and $D^2\subset\mathcal{E}_7(g^t)$, $D\subset(G-D\circ g^t)\cap(G-g^t\circ D)$, $D\cap(D\circ g^t\cup g^t\circ D)=\phi$, $D\circ g^t\cup g^t\circ D\subset G-D$, $D\subset\mathcal{E}_7(g^t)\circ D\cap D\circ\mathcal{E}_7(g^t)$.

Theorem 10. $B(g)=G/\mathcal{E}_7(g^t)\cap[\cup\{G/\delta\mid\delta\in E[G)\ ,\ g\subset\delta\}]$ is a subset of the intersection of strange attractor set for g and strange panattractor set for g^t.

Theorem 11. The BC^2S nature described by $D\circ g\subset D$ or $D\circ g\cap D=\phi$ is conservative for confinement, embodiment and PM microscopy.

Theorem 12. Let $f\subset G\times H$ be bicovering, $G=\cup G_i(d\delta_1(f\circ f^{-1}))$, then for $i\neq j$, we have $G_i\circ f\cap G_j\circ f=\phi$.

Theorem 13. Let $f\subset G\times H$, $\theta\subset G^2$, $\theta=\theta^{-1}$, $\delta\in E_5[H)$, $\delta\subset\mathcal{E}_7(f'(\theta))$, $H=\cup H_i(d\delta)$, $\varphi_i=f\cap(f\circ H_i)\times H_i$, then $\varphi_i: f\circ H_i\longrightarrow H_i$.

REFERENCES

[1] Yu Qing, Pansystems Methodology: Background, Contents and Meaning (in Chinese), Exploration of Nature, 1(1984).

[2] Wu Xuemou, Pansystems Methodology: Concepts, Theorems and Applications (I)-(VIII), Science Exploration, 1,2,4(1982), 1,4(1983), 1,4(1984).

[3] Wu Xuemou, Investigation and Applications of Pansystems Recognition Theory and Pansystems-Operations Research of Large Scale System (I), Applied Math. and Mechanics, 1(1984).

[4] Wu Xuemou, Fixed Pansystems Theorems and Pansystems Catastrophe Analysis of Panweighted Network, Math. Res. and Exposition, 1(1984).

[5] Wu Xuemou, Pansystems Methodology and Pansystems Catastrophe Analysis (in Chinese), Chinese Symposium on Bifurcation, Catastrophe and Stability, 1983.

[6] Wu Xuemou, Pansystems Metatheory of Ecology, Medicine and Diagnosis (I)-(IV) (in Chinese), Exploration of Nature, 2,3(1983), 1(1984), 2(1985).

[7] Gao Longying, Wang Shuji, Pansymmetry and Fixed Pansystems Theorems, Applied Math. and Mechanics, 5(1984).

[8] Li Guihua, The Characteristic Describing of the Structure and Quantity in Fixed Pansystems Theorems, Applied Math. and Mechanics, 6(1984).

[9] Wu Shouzhi, Wu Xuemou, Generalized Principle of Optimality in Pansystems Network Analysis, Kybernetes, 4(1984).

[10] Chen Beifang, Wu Xuemou, Some Pansystems Models of General Equilibrium in Economics, Kybernetes, 1(1985).

[11] Wu Xuemou, Pansystems Methodology: A Transfield Investigation of Generalized System-Transformation-Symmetry, Fuzzy Information and Decision Processes, M.M.Gupta and E.Sanchez(eds.), North-Holland Publishing Company, 1982.

[12] Wu Xuemou, Pansystems Analysis: Some New Investigations of Logic, Observocontrollability and Fuzziness, Journal of Huazhong I.T.(English Edition), 1(1981).