THE EIGHT NEW KINDS OF HYPERTOPOLOGIES

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Introduction

As a basis theory of fuzzy sets,in literatures (1),(2), Prof.Wang Pei-Zhuang introduced eight kinds of hypertopologies through latticed topologies.These eight kinds of hypertopologies include the hypertopologies provided in (4),(5),(6),as well as several new hypertopologies not defined before.In literature (3),the convergences of these hypertopologies and their applications were studied.In this paper,on the basis of literatures (1),(2),(3),we combine eight new kinds of hypertopologies from four kinds of hypertopologies which has one-way convergence in literatures (1),(2).And give the laws of convergence for each of them,and give the rigion limit points of set-net in,and make a comparison between all sixteen kinds of hypertopologies.The homeomorphic equivalent classes of all sixteen kinds of hypertopologies is a distributive lattice.

Keywords: Hypertopology,Hyperspace,Set-net,Convergence.

In this paper,we adopt essentially the notations in (1), (3).For convenience latter,we begin by listing some pricipal conventions and notations:

CONVENTIONS:

(1) A set-net $\{A_\alpha\}_{\alpha\in D}$ in X is a net in the power set $\mathcal{P}(x)$ of X;

(2) Let $(X,\mathcal{G})$ be a topological space, $\mathcal{P}(X)$ denotes the power set of X. $\mathcal{F}(X)$ denotes the collection of all closed subsets of X, $\mathcal{G}(X)$ denotes the collection of all open subsets of X, $\mathcal{H}(X)$ denotes the collection of all clopen subsets of X. F,G,H ranges over closed,open,clopen subsets of X.

(3) For each $A\in\mathcal{P}(X)$,let $\dot{A}=\{B\in\mathcal{P}(X)\mid B\supset A\}$,abd $\underset{\cdot}{A}=\{B\in\mathcal{P}(X)\mid B\subset A\}$,and for each $\mathscr{D}\subset\mathcal{P}(X)$,let $\mathscr{D}^{\backslash c}=\{A^c=X\setminus A\mid A\in\mathscr{D}\}$.

[illegible] here are definition as follow in literatyres(1),(2),(3):

$T_{10}(\mathcal{P}(X))$ denotes the hypertopology in X whose base is $\mathcal{G}\triangleq$ $\{[illegible]\}$ [illegible] $\{A_d\}_{d\in D}$ converges to A in $(\mathcal{P}(X).T_{10}(\mathcal{P}(X)))$ will be denoted by [illegible].

$T_{01}(\mathcal{P}(X))$ denotes the hypertopology in X whose base is [illegible] $\triangleq$ [illegible]. Net $\{A_d\}_{d\in D}$ converges to A in $(\mathcal{P}(X),T_{01}(\mathcal{P}(X)))$ will be denoted by [illegible] A.

$T_{20}(\mathcal{P}(X))$ denoted the hypertopology in X whose subbase is $(\mathcal{G})\triangleq$ [illegible] $\mathcal{P}(X)$ G G $\mathcal{G}\}\cup\{\mathcal{P}(X)\}$. And $\{A_d\}_{d\in D}$ converges to A in $(\mathcal{P}(X),T_{20}(\mathcal{P}(X)))$ will be denoted by [illegible] A.

$T_{02}(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is $(\mathcal{F})\triangleq$ $\mathcal{P}(X)$ F F [illegible] $\}\cup\{\mathcal{P}(X)$, and $\{A_d\}_{d\in D}$ converges to A in $(\mathcal{P}(X),T_{02}(\mathcal{P}(X)))$ will be denoted as [illegible] A.

T [illegible] $(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is [illegible]. That [illegible] converges to A in $(\mathcal{P}(X),T_{11}(\mathcal{P}(X)))$ will be denoted [illegible] A.

T [illegible] $(X))$ denotes the hypertopology in X whose subbase is [illegible]. And [illegible] converges to A in $(\mathcal{P}(X),T_{12}(\mathcal{P}(X)))$ will be denoted as [illegible] A.

T [illegible] $(X))$ denotes the hypertopology in X whose subbase is [illegible]. And [illegible] converges to A in $(\mathcal{P}(X),T_{21}(\mathcal{P}(X)))$ will be denoted as [illegible] A.

T [illegible] $(X))$ denotes the hypertopology in X whose subbase is [illegible]. And [illegible] converges to A in $(\mathcal{P}(X),T_{22}(\mathcal{P}(X)))$ will be denoted as [illegible] A.

[illegible] Let [illegible] $\subset\mathcal{P}(X)$, $T_\omega(\mathcal{D})$ indicates the induced topology on $\mathcal{D}$ of [illegible] $))$, where $\omega\in\{10,01,20,02,11,12,21,22\}$.

[illegible] $T_\omega(\mathcal{D})$ denotes the space $(\mathcal{D},T_\omega(\mathcal{D}))$, and sometimes denotes the topology $T_\omega(\mathcal{D})$.

[illegible] Let $\{A_d\}_{d\in D}$ be a set-net in topological space$(X,\mathcal{G})$.

Define

$\mathrm{Lim}_D A_d=\{x\in X|$ For evry neighbourhood U(x) of x, there exists a subnet [illegible] of net $\{A_d\}_{d\in D}$ such that $U(x)\cap A_{d'}\neq\emptyset$ for every $d'\in D'\}$.

$\underline{\mathrm{Lim}}_D A_d=\{x\in X|$ for every neighbourhood U(X) of x, there exists $d_0\in D$ such that $U(x)\cap A_d\neq\emptyset$ for every $d\geqslant d_0\}$.

$\overline{\overline{\mathrm{Lim}}}_D A_d=\{x\in X|$ There exists a neighbourhood U(x) of x and a subnet [illegible] D' such that $U(x)\subset A_{d'}$ for every $d'\in D'\}$.

$\underline{\underline{\mathrm{Lim}}}_D A_d=\{x\in X|$ there exists a neighbourhood U(x) of x and $d_0\in D$

such that $U(x) \subset$ [illegible] for every d [illegible] d_0.

(3)Let T and T' be twe hypertopologies in $(X, \mathcal{G})$.They are said to be dual if $T' = \{G^c$ [illegible] $\}$ =T. T is said to be symmetrical if T and T' are dual. T and T' are dual if and only if they have a pair of dual subbases.

§ Definition of eight new kinds of hypertopologies

Definition 1.' Let $(X, \mathcal{G})$ be a topological space , $(\mathcal{P}(X), T_\omega(\mathcal{P}(X)))$ (ω= 03,30,13,31 23,32,33,00) are hyperspaces defined as follows:

$T_{30}(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is [illegible] .and $\{A_\alpha\}_{\alpha\in D}$ converges to A in $(\mathcal{P}(X), T_{30}(\mathcal{P}(X)))$ will be denoted as A_α [illegible] A.

[illegible] $(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is [illegible] .and $\{A_\alpha\}_{\alpha\in D}$ converges to A in $(\mathcal{P}(X), T$ [illegible] $(\mathcal{P}(X)))$ will be denoted as $A_\alpha \twoheadrightarrow A$.

$T_{13}(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is [illegible] $)^c$,and $\{A_\alpha\}_{\alpha\in D}$ converges to A in $(\mathcal{P}(X), T_{13}(\mathcal{P}(X)))$ will be denoted as A_α [illegible] A.

$T_{31}(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is $\mathcal{G}$ [illegible] $)^c$,and $\{A_\alpha\}_{\alpha\in D}$ converges to A in $(\mathcal{P}(X), T_{31}(\mathcal{P}(X)))$ will be denoted as A_α [illegible] A.

$T_{23}(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is [illegible] $\cup (\mathcal{F})^c$,and $\{A_\alpha\}_{\alpha\in D}$ converges to A in $(\mathcal{P}(X), T_{23}(\mathcal{P}(X)))$ will be denoted as A_α [illegible] A.

$T_{32}(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is [illegible] $(\mathcal{G})^c$,and $\{A_\alpha\}_{\alpha\in D}$ converges to A in $(\mathcal{P}(X), T_{32}(\mathcal{P}(X)))$ will be denoted as A_α [illegible] A.

$T_{33}(\mathcal{P}(X))$ denotes the hypertopology in X whose subbase is $\mathcal{G} \cup \mathcal{F} \cup (\mathcal{G})^c \cup (\mathcal{F})^c$,and $\{A_\alpha\}_{\alpha\in D}$ converges to A in $(\mathcal{P}(X), T_{33}(\mathcal{P}(X)))$ will be denoted as $A_\alpha \twoheadrightarrow A$.

For perfectness and harmoniousness in mathematics,we must add indiscrete topology $T_{00}(\mathcal{P}(X))$ whose open set is $\mathcal{P}(X)$ and $\emptyset$.

These eight kinds of hypertopologies with those in literretures (1),(2) are sixteen kinds of hypertopologies. It is obvious that it can not produs any new hypertopology to combine these hypertopologies.

PROPOSITION1.1 $T_{30}(\mathcal{P}(X))$ and $T_{03}(\mathcal{P}(X))$ are dual, $T_{23}(\mathcal{P}(X))$ and $T_{32}(\mathcal{P}(X))$ are dual, $T_{13}(\mathcal{P}(X))$ and $T_{31}(\mathcal{P}(X))$ are dual; $T_{33}(\mathcal{P}(X))$ is symmetry.

Proof. Because a subbase of $T_{30}(\mathcal{P}(X))$ is $\underset{\cdot}{\mathcal{G}} \cup (\dot{\mathcal{G}})^{\backslash c}$, and a subbase of $T_{03}(\mathcal{P}(X))$ is $\dot{\mathcal{F}} \cup (\underset{\cdot}{\mathcal{F}})^{\backslash c}$. Since $(\underset{\cdot}{\mathcal{G}})^{\backslash c} = \dot{\mathcal{F}}$, $((\dot{\mathcal{G}})^{\backslash c})^{\backslash c} = (\underset{\cdot}{\mathcal{F}})^{\backslash c}$, that is, a subbase of $T_{30}(\mathcal{P}(X))$ and a subbase of $T_{03}(\mathcal{P}(X))$ are dual. Then $T_{30}(\mathcal{P}(X))$ and $T_{03}(\mathcal{P}(X))$ are dual. The proofs of the rest are analogous.

PROPOSITION 1.2 $A_d \rightharpoonup\!\!\rightharpoonup A \Leftrightarrow A_d \dashrightarrow A$and $A_d \rightharpoonup\!\!\rightharpoonup A \Leftrightarrow$ $A_d \rightharpoonup\!\!\rightharpoonup A$ and $A_d \rightharpoondown A \Leftrightarrow A_d \dashrightarrow A$ and $A_d \dashrightarrow\!\!\rightharpoonup A \Leftrightarrow A_d \rightharpoonup\!\!\rightharpoonup A$ and $A_d \longrightarrow A$.

Proof. We only prove $A_d \rightharpoonup\!\!\rightharpoonup A \Leftrightarrow A_d \longrightarrow A$ and $A_d \rightharpoonup\!\!\rightharpoonup A$.

"$\Rightarrow$" In $(\mathcal{P}(X), T_{11}(\mathcal{P}(X)))$ any base neighborhood of A $\underset{\cdot}{G} \cap \dot{F}$, in $(\mathcal{P}(X), T_{02}(\mathcal{P}(X)))$ any base neighborhood $(\underset{\cdot}{F})^{\backslash c}$ of A. Then $\underset{\cdot}{G} \cap \dot{F} \cap (\underset{\cdot}{F})^{\backslash c}$ is a base neighborhood of A in $(\mathcal{P}(X), T_{13}(\mathcal{P}(X)))$, because $A_d \rightharpoonup\!\!\rightharpoonup A$, then $\exists\, d_0 \in D, \forall d \geqslant d_0, d \in D, A_d \in \underset{\cdot}{G} \cap \dot{F} \cap (\underset{\cdot}{F})^{\backslash c} \Rightarrow \forall d \geqslant d_0$, $A_d \in \underset{\cdot}{G} \cap \dot{F}$ and $A_d \in (\underset{\cdot}{F})^{\backslash c} \Rightarrow A_d \longrightarrow A$ and $A_d \rightharpoonup\!\!\rightharpoonup A$.

"$\Leftarrow$" In $(\mathcal{P}(X), T_{13}(\mathcal{P}(X)))$ any base neighborhood of A $\underset{\cdot}{G} \cap \dot{F} \cap (\underset{\cdot}{F})^{\backslash c}$, then $\underset{\cdot}{G} \cap \dot{F}$ is base neighborhood of A in $(\mathcal{P}(X), T_{11}(\mathcal{P}(X)))$, becaus $A_d \longrightarrow A$, then $\exists\, d_0' \in D, \forall d \geqslant d_0', A_d \in \underset{\cdot}{G} \cap \dot{F}$; and $(\underset{\cdot}{F})^{\backslash c}$ is a base neighborhood of A in $(\mathcal{P}(X). T_{02}(\mathcal{P}(X)))$, becaus $A_d \rightharpoonup\!\!\rightharpoonup A$, then $\exists\, d_0'' \in D$, $\forall d \geqslant d_0'', A_d \in (\underset{\cdot}{F})^{\backslash c}$. Let $d_0 \geqslant d_0', d_0 \geqslant d_0'', d_0 \in D$, then $\forall d \geqslant d_0, d \in D, A_d \in \underset{\cdot}{G} \cap \dot{F} \cap (\underset{\cdot}{F})^{\backslash c}$, therefore $A_d \rightharpoonup\!\!\rightharpoonup A$.

The rest of the proof is analogous, omitted.

PROPOSITION 1.3 $A_d \twoheadrightarrow A \Leftrightarrow A_d \dashrightarrow A$ and $A_d \longrightarrow A \Leftrightarrow$ $A_d \twoheadrightarrow A$ and $A_d \rightharpoonup A \Leftrightarrow A_d \longrightarrow$ Aand $A_d \dashrightarrow A \Leftrightarrow A_d \twoheadrightarrow A$ and $A_d \rightharpoonup A$.

PROPOSITION 1.4 $A_d \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A \Leftrightarrow A_d \dashrightarrow A$ and $A_d \rightharpoonup\!\!\rightharpoonup A \Leftrightarrow$ $A_d \twoheadrightarrow A$ and $A_d \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A \Leftrightarrow A_d \rightharpoonup A$ and $A_d \twoheadrightarrow A$.

PROPOSITION 1.5 $A_d \twoheadrightarrow A \Leftrightarrow A_d \rightharpoonup\!\!\rightharpoonup A$ and $A_d \twoheadrightarrow A \Leftrightarrow$ $A_d \rightharpoonup\!\!\rightharpoonup A$ and $A_d \twoheadrightarrow A \Leftrightarrow A_d \twoheadrightarrow A$ and $A_d \longrightarrow A$.

PROPOSITION1.6 $A_d \twoheadrightarrow\!\!\rightarrow A \Leftrightarrow A_d \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A$ and $A_d \twoheadrightarrow A \Leftrightarrow$ $A_d \dashrightarrow A$ and $A_d \twoheadrightarrow A \Leftrightarrow A_d \longrightarrow A$ and $A_d \twoheadrightarrow\!\!\rightarrow A \Leftrightarrow A_d \rightharpoonup\!\!\rightharpoonup A$ and $A_d \twoheadrightarrow\!\!\rightarrow A \Leftrightarrow$ $A_d \twoheadrightarrow A$ and $A_d \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A \Leftrightarrow A_d \dashrightarrow A$ and $A_d \twoheadrightarrow A \Leftrightarrow A_d \rightharpoonup A$ and $A_d \twoheadrightarrow\!\!\rightarrow A$.

Remark. $A_d \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A$ is different from $A_d \longrightarrow A$ and $A_d \rightharpoonup\!\!\rightharpoonup A$.
$A_d \twoheadrightarrow A$ is different from $A_d \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A$ and $A_d \longrightarrow A$.
$A_d \twoheadrightarrow\!\!\rightarrow A$ is different from $A_d \twoheadrightarrow A$ and $A_d \rightharpoonup A$.
$A_d \twoheadrightarrow\!\!\rightarrow A$ is different from $A_d \longrightarrow A$ and $A_d \twoheadrightarrow A$.

Therefore, it will produce mistakes to "separat" or "merge" the symbol of convergences freely.

§ 2 The convergence laws and the regions of its limits

THEOREM 2.1 (The test of existence of limits)

Let $(X,\mathcal{T})$ be a compact Hausdorff space and $\{A_\alpha\}_{\alpha\in D}$ be a set-net in X. Then

(1) $A_\alpha \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A \Rightarrow \overline{\lim}_D A_\alpha \subset \bar{A}$ and $\overline{\overline{\lim}}_D A_\alpha \subset A$; conversely, $\overline{\lim}_D A_\alpha \subset A \Rightarrow A_\alpha \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A$. Particuly, in hyperspace $(\mathcal{F}, T_{30}(\mathcal{F}))$, net $\{F_\alpha\in\mathcal{F}\}_{\alpha\in D}$ converges to $F\in\mathcal{F}$, i.e., $F_\alpha \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup F \iff \overline{\lim}_D F_\alpha \subset F$. $(\mathcal{F}, T_{30}(\mathcal{F}))$ and $(\mathcal{F}, T_{10}(\mathcal{F}))$ are homeomorphism.

(2) $A_\alpha \rightharpoondown\!\!\rightharpoondown\!\!\rightharpoondown A \Rightarrow A^\circ \subset \underline{\underline{\lim}}_D A_\alpha$ and $A \subset \underline{\lim}_D A_\alpha$; conversely, $\underline{\lim}_D A_\alpha \supset A \Rightarrow A_\alpha \rightharpoondown\!\!\rightharpoondown\!\!\rightharpoondown A$. particularly, in hyperspace $(\mathcal{G}, T_{03}(\mathcal{G}))$, net $\{G_\alpha\in\mathcal{G}\}_{\alpha\in D}$ converges to $G\in\mathcal{G}$, i.e., $G_\alpha \rightharpoondown\!\!\rightharpoondown\!\!\rightharpoondown G \iff \underline{\underline{\lim}}_D G_\alpha \supset G$. $(\mathcal{G}, T_{03}(\mathcal{G}))$ and $(\mathcal{G}, T_{01}(\mathcal{G}))$ are homeomorphism.

(3) $A_\alpha \twoheadrightarrow A \Rightarrow A^\circ \subset \underline{\underline{\lim}}_D A_\alpha$ and $\bar{A}=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha$; conversely, $A=\overline{\lim}_D A_\alpha=\underline{\lim}_D A_\alpha \Rightarrow A_\alpha \twoheadrightarrow A$. Particularly, in hyperspace $(\mathcal{H}, T_{13}(\mathcal{H}))$, net $\{H_\alpha\in\mathcal{H}\}_{\alpha\in D}$ converges to $H\in\mathcal{H}$, i.e.,

$H_\alpha \twoheadrightarrow H \iff H=\overline{\lim}_D H_\alpha=\underline{\underline{\lim}}_D H_\alpha$.

(4) $A_\alpha \twoheadrightarrow A \Rightarrow A^\circ=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha$ and $\overline{\lim}_D A_\alpha \subset \bar{A}$; conversely, $A=\overline{\lim}_D A_\alpha=\underline{\underline{\lim}}_D A_\alpha \Rightarrow A_\alpha \twoheadrightarrow A$. particularly, in hyperspace $(\mathcal{H}, T_{31}(\mathcal{H}))$,

$H_\alpha \twoheadrightarrow H \iff H=\overline{\lim}_D H_\alpha=\underline{\underline{\lim}}_D H_\alpha$.

(5) $A_\alpha \twoheadrightarrow A \Rightarrow A^\circ=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha$ and $A \subset \underline{\lim}_D A_\alpha$; conversely, $A=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha \Rightarrow A_\alpha \twoheadrightarrow A$. Particularly, in hyperspace $(\mathcal{G}, T_{23}(\mathcal{G}))$,

$G_\alpha \twoheadrightarrow G \iff G=\underline{\underline{\lim}}_D G_\alpha=\overline{\overline{\lim}}_D G_\alpha$.

(6) $A_\alpha \twoheadrightarrow A \Rightarrow \overline{\overline{\lim}}_D A_\alpha \subset A$ and $\bar{A}=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha$; conversely, $A=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha \Rightarrow A_\alpha \twoheadrightarrow A$. Particularly, in hyperspace $(\mathcal{F}, T_{32}(\mathcal{F}))$,

$F_\alpha \twoheadrightarrow F \iff F=\underline{\lim}_D F_\alpha=\overline{\lim}_D F_\alpha$.

(7) $A_\alpha \twoheadrightarrow A \Rightarrow A^\circ=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha$ and $\bar{A}=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha$; conversely, $A=\overline{\lim}_D A_\alpha=\underline{\lim}_D A_\alpha \Rightarrow A_\alpha \twoheadrightarrow A$. Particularly, in hyperspace $(\mathcal{H}, T_{33}(\mathcal{H}))$, $H_\alpha \twoheadrightarrow H \iff H=\overline{\lim}_D H_\alpha=\underline{\underline{\lim}}_D H_\alpha$.

Proof. (1) $A_\alpha \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A \iff A_\alpha \rightharpoonup A$ and $A_\alpha \rightharpoondown A$ (definition) $\Rightarrow \overline{\lim}_D A_\alpha \subset \bar{A}$ and $\overline{\overline{\lim}}_D A_\alpha \subset A$ (In literature(3)theorem2.1).

conversely, $\overline{\lim}_D A_\alpha \subset A \Rightarrow A_\alpha \rightharpoonup A$ and $\overline{\overline{\lim}}_D A_\alpha \subset \overline{\lim}_D A_\alpha \subset A \Rightarrow A_\alpha \rightharpoonup A$ and $A_\alpha \rightharpoondown A$(In literature(3)theorem2.1) $\Rightarrow A_\alpha \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup A$. In hyperspace $(\mathcal{F}, T_{30}(\mathcal{F}))$, because $\bar{F}=F$, so $\overline{\overline{\lim}}_D F_\alpha \subset \overline{\lim}_D F_\alpha \subset \bar{F}=F$, then $F_\alpha \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup F \iff \overline{\lim}_D F_\alpha \subset F$. Because in hyperspace $(\mathcal{F}, T_{10}(\mathcal{F}))$, $F_\alpha \rightharpoonup F \iff \overline{\lim}_D F_\alpha \subset F$, hence $F_\alpha \rightharpoonup\!\!\rightharpoonup\!\!\rightharpoonup F$ (in $(\mathcal{F}, T_{30}(\mathcal{F}))$) $\iff F_\alpha \rightharpoonup F$ (in $(\mathcal{F}, T_{10}(\mathcal{F}))$). i.e., $(\mathcal{F}, T_{30}(\mathcal{F}))$ and $(\mathcal{F}, T_{10}(\mathcal{F}))$ are [illegible].

The proofs of (2) —(7) are analogous.

THEOREM 1 (Determinations for regions of limit points)

Let X be a compact Hausdorff space.Then

(1) In hyperspace $(\mathcal{P}(X), T_{30}(\mathcal{P}(X)))$,the set$\{A\in\mathcal{P}(X) \mid A_\alpha \twoheadrightarrow A\}$ consisting of all limit points of net $\{A_\alpha\}_{\alpha\in D}$ is included in the set $\{A\in\mathcal{P}(X) \mid \overline{\lim}_D A_\alpha \subset \bar{A}$ and $\overline{\overline{\lim}}_D A_\alpha \subset A\}$ and includes the set $\{A\in\mathcal{P}(X) \mid \overline{\overline{\lim}}_D A_\alpha \subset A\}$, i.e., $\{A\in\mathcal{P}(X) \mid \overline{\overline{\lim}}_D A_\alpha \subset A\} \subset \{A\in\mathcal{P}(X) \mid A_\alpha \twoheadrightarrow A\} \subset$ $\subset \{A\in\mathcal{P}(X) \mid \overline{\lim}_D A_\alpha \subset \bar{A}$ and $\overline{\overline{\lim}}_D A_\alpha \subset A\}$ .In general,theinclusions are proper.But in hyperspace $(\mathcal{F}, T_{30}(\mathcal{F}))$,The set $\{F\in\mathcal{F} \mid F_\alpha \twoheadrightarrow F\}$ consisting of all limit points of net$\{F_\alpha\}_{\alpha\in D}$ in $\mathcal{F}$ equals the set $\{F\in\mathcal{F} \mid \overline{\overline{\lim}}_D F_\alpha \subset F\} = (\overline{\overline{\lim}}_D F_\alpha)^{\cdot}\cap\mathcal{F}$.

(2) In hyperspace $(\mathcal{P}(X), T_{03}(\mathcal{P}(X)))$,the set $\{A\in\mathcal{P}(X) \mid A_\alpha \twoheadrightarrow A\}$ consisting of all limit point of net$\{A_\alpha\}_{\alpha\in D}$ is included in the set$\{A\in\mathcal{P}(X) \mid A^\circ \subset \underline{\underline{\lim}}_D A_\alpha$ and $A \subset \underline{\lim}_D A_\alpha\}$and includes the set$\{A\in\mathcal{P}(X) \mid A \subset \underline{\underline{\lim}}_D A_\alpha\}$,i.e.,$\{A\in\mathcal{P}(X) \mid A \subset \underline{\underline{\lim}}_D A_\alpha\} \subset \{A\in\mathcal{P}(X) \mid A_\alpha \twoheadrightarrow A\} \subset \{A\in\mathcal{P}(X) \mid A^\circ \subset \underline{\underline{\lim}}_D A_\alpha$ and $A \subset \underline{\lim}_D A_\alpha\}$ .In general, the inclusions are proper.But in hyperspace $(\mathcal{G}, T_{03}(\mathcal{G}))$,the set$\{G\in\mathcal{G} \mid G_\alpha \twoheadrightarrow G\}$ consisting of all limit points of net $\{G_\alpha\}_{\alpha\in D}$ equals the set $\{G\in\mathcal{G} \mid G \subset \underline{\underline{\lim}}_D G_\alpha\} = (\underline{\underline{\lim}}_D G_\alpha)_{\cdot}\cap\mathcal{G}$.

(3) In hyperspace $(\mathcal{P}(X), T_{13}(\mathcal{P}(X)))$,the set$\{A\in\mathcal{P}(X) \mid A_\alpha \twoheadrightarrow A\}$ consisting of all limit point of net$\{A_\alpha\}_{\alpha\in D}$ is included in the set$\{A\in\mathcal{P}(X) \mid A^\circ \subset \underline{\underline{\lim}}_D A_\alpha$ and $\bar{A} = \underline{\lim}_D A_\alpha = \overline{\lim}_D A_\alpha\}$ and includes the set$\{A\in\mathcal{P}(X) \mid A = \overline{\overline{\lim}}_D A_\alpha = \underline{\underline{\lim}}_D A_\alpha\}$ .In general,the inclusion are proper. But in hyperspace $(\mathcal{H}, T_{13}(\mathcal{H}))$,the set$\{H\in\mathcal{H} \mid H_\alpha \twoheadrightarrow H\}$ consisting of all limit points of net$\{H_\alpha\}_{\alpha\in D}$ in $\mathcal{H}$ equals the set$\{H\in\mathcal{H} \mid H = \overline{\overline{\lim}}_D H_\alpha = \underline{\underline{\lim}}_D H_\alpha\}$.It is obvious that $(\mathcal{H}, T_{13}(\mathcal{H}))$ in a Hausdorff space.

(4) In hyperspace $(\mathcal{P}(X), T_{31}(\mathcal{P}(X)))$,the set$\{A\in\mathcal{P}(X) \mid A_\alpha \twoheadrightarrow A\}$ consisting of all limit points of net$\{A_\alpha\}_{\alpha\in D}$ is included in the set$\{A\in\mathcal{P}(X) \mid A^\circ = \underline{\underline{\lim}}_D A_\alpha = \overline{\overline{\lim}}_D A_\alpha$ and $\overline{\lim}_D A_\alpha \subset \bar{A}\}$ and includes the set $\{A\in\mathcal{P}(X) \mid A = \overline{\overline{\lim}}_D A_\alpha = \underline{\underline{\lim}}_D A_\alpha\}$ .In general, the inclusions are proper.But in hyperspace $(\mathcal{H}, T_{31}(\mathcal{H}))$,the set$\{H\in\mathcal{H} \mid H_\alpha \twoheadrightarrow H\}$ consisting of all limit points of net$\{H_\alpha\}_{\alpha\in D}$ in $\mathcal{H}$ equals the set$\{H\in\mathcal{H} \mid H = \overline{\overline{\lim}}_D H_\alpha = \underline{\underline{\lim}}_D H_\alpha\}$.

It is obvious that $(\mathcal{H}, T_{31}(\mathcal{H}))$ is a Hausdorff space.

(5) Inhyperspace $(\mathcal{P}(X), T_{23}(\mathcal{P}(X)))$,the set$\{A\in\mathcal{P}(X) \mid A_\alpha \twoheadrightarrow A\}$

consisting of all limit points of net$\{A_\alpha\}_{\alpha\in D}$ is included in the set$\{A\in\mathcal{P}(X)\mid A^\circ=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha$ and $A\subset\underline{\lim}_D A_\alpha\}$ and includes the set$\{A\in\mathcal{P}(X)\mid A=\underline{\underline{\lim}}_D A_\alpha=\overline{\lim}_D A_\alpha\}$. In general, the inclusions are proper. But in hyperspace $(\mathcal{G},T_{23}(\mathcal{G})))$, the set$\{G\in\mathcal{G}\mid G_\alpha\twoheadrightarrow G\}$ consisting of all limit points of net$\{G_\alpha\}_{\alpha\in D}$ equals the set $\{G\in\mathcal{G}\mid G=\underline{\underline{\lim}}_D G_\alpha=\overline{\overline{\lim}}_D G_\alpha\}$. It is obvious that $(\mathcal{G},T_{23}(\mathcal{G}))$ is a Hausdorff space.

(6) In hyperspace $(\mathcal{P}(X),T_{32}(\mathcal{P}(X)))$, the set$\{A\in\mathcal{P}(X)\mid A_\alpha\twoheadrightarrow A\}$ consisting of all limit points of net$\{A_\alpha\}_{\alpha\in D}$ is included in the set$\{A\in\mathcal{P}(X)\mid \overline{\overline{\lim}}_D A_\alpha\subset A$ and $\bar{A}=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha\}$ and includes the set$\{A\in\mathcal{P}(X)\mid A=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha\}$. In general, the inclusions are proper. But in hyperspace $(\mathcal{F},T_{32}(\mathcal{F}))$, the set $\{F\in\mathcal{F}\mid F_\alpha\twoheadrightarrow F\}$ consisting of all limit points of net$\{F_\alpha\}_{\alpha\in D}$ in $\mathcal{F}$ equals the set$\{F\in\mathcal{F}\mid F=\underline{\lim}_D F_\alpha=\overline{\lim}_D F_\alpha\}$. It is obvious that $(\mathcal{F},T_{32}(\mathcal{F}))$ is a Hausdorff space.

(7) In hyperspace $(\mathcal{P}(X),T_{33}(\mathcal{P}(X)))$, the set$\{A\in\mathcal{P}(X)\mid A_\alpha\twoheadrightarrow A\}$ consisting of all limit points of net$\{A_\alpha\}_{\alpha\in D}$ is included in the set$\{A\in\mathcal{P}(X)\mid A^\circ=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha$ and $\bar{A}=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha\}$ and includes the set$\{A\in\mathcal{P}(X)\mid A=\overline{\lim}_D A_\alpha=\underline{\underline{\lim}}_D A_\alpha\}$. In general, the inclusions are proper. But in hyperspace $(\mathcal{H},T_{33}(\mathcal{H}))$, the set $\{H\in\mathcal{H}\mid H_\alpha\twoheadrightarrow H\}$ consisting of all limit points of net$\{H_\alpha\}_{\alpha\in D}$ in $\mathcal{H}$ equals the set$\{H\in\mathcal{H}\mid H=\overline{\lim}_D H_\alpha=\underline{\underline{\lim}}_D H_\alpha\}$. It is obvious that $(\mathcal{H},T_{33}(\mathcal{H}))$ is Hausdorff space.

COROLLARY 2. Let X be a compact Hausdorff space. Then

(1) If a net$\{A_\alpha\}_{\alpha\in D}$ in $(\mathcal{P}(X),T_{13}(\mathcal{P}(X)))$ is convergent, then it has unique closed (in X) limit point: $A=\overline{\lim}_D A_\alpha=\underline{\lim}_D A_\alpha$.

(2) If a net$\{A_\alpha\}_{\alpha\in D}$ in $(\mathcal{P}(X),T_{31}(\mathcal{P}(X)))$ is convergent, then it has unique open (in X) limit point: $A=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha$.

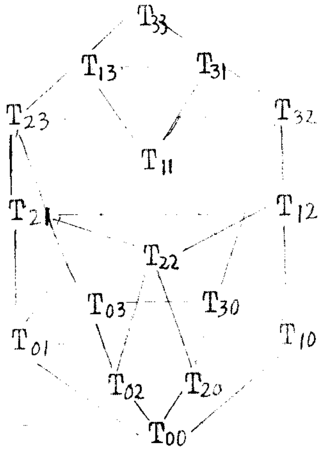
(3) If a net$\{A_\alpha\}_{\alpha\in D}$ in $(\mathcal{P}(X),T_{23}(\mathcal{P}(X)))$ is convergent, then it has unique open (in X) limit point: $A=\underline{\underline{\lim}}_D A_\alpha=\overline{\overline{\lim}}_D A_\alpha$.

(4) If a net$\{A_\alpha\}_{\alpha\in D}$ in $(\mathcal{P}(X),T_{32}(\mathcal{P}(X)))$ is convergent, then it has unique closed (in X) limit point: $A=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha$.

(5) If a net$\{A_\alpha\}_{\alpha\in D}$ in $(\mathcal{P}(X),T_{33}(\mathcal{P}(X)))$ is convergent, then it has unique open (in X) limit point: $A=\overline{\overline{\lim}}_D A_\alpha=\underline{\underline{\lim}}_D A_\alpha$. (or it has unique closed (in X) limit point: $A=\underline{\lim}_D A_\alpha=\overline{\lim}_D A_\alpha$).

§3 The comparison between sixteen kinds of Hypertopologies

THEOREM 3.1 Let X be a compact Hausdorff space. Then, in set $\mathcal{P}(X)$, the relation between the sixteen kinds of hypertopologies indicated in figue 1.(There the result that comparsion among all eight kinds of hypertopologies in literatures (3) is quoted.)



Figue 1.
T_ω indicates
$T_\omega(\mathcal{P}(X))$

There the topologies upper are finer than the lower (at the same line),and two topologies at the same level line (dotted line) are homeomorphic in following map:

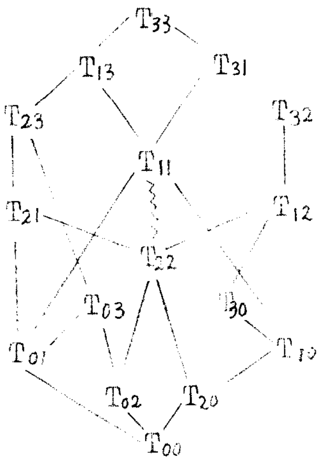
$$f: \mathcal{P}(X) \to \mathcal{P}(X)$$
$$A \mapsto A^c$$

Proof. Because $A_\alpha \twoheadrightarrow A \Longleftrightarrow A_\alpha \not\to A$ and $A_\alpha \twoheadrightarrow A \Rightarrow A_\alpha \twoheadrightarrow A$, $T_{33}(\mathcal{P}(X))$is finer than $T_{31}(\mathcal{P}(X))$.

Since $A_\alpha \to A \Rightarrow A_\alpha \not\twoheadrightarrow A$ (In literature (3) theorem 3.1), $A_\alpha \twoheadrightarrow A \Rightarrow A_\alpha \to A$ and $A_\alpha \twoheadrightarrow A \Rightarrow A_\alpha \to A$ and $A_\alpha \not\twoheadrightarrow A \Rightarrow A_\alpha \twoheadrightarrow A$,so $T_{31}(\mathcal{P}(X))$ is finer than $T_{32}(\mathcal{P}(X))$.

The rest of the proof is analogous, omitted.

THEOREM 3.2 Let X be a compact Hausdorff space. Then, in the collection $\mathcal{F}$ consisting of all closed subsets of X, the relation between the sixteen kinds of hypertopologies indicated in figue 2.There, thetopologies upper are finer than the lower, and if $T_{11}(\mathcal{F})$ is compact, so is [illegible].



Figue 2. T_ω indicates $T_\omega(\mathcal{F})$

Proof. In $(\mathcal{F}, T_{10}(\mathcal{F}))$,
$F_\alpha \to F \Longleftrightarrow \overline{\lim}_D F_\alpha \subset \overline{F} = F \Rightarrow$
$\underline{\lim}_D F_\alpha \subset \overline{\lim}_D F_\alpha \subset F \Rightarrow F_\alpha \to F$,
so $T_{10}(\mathcal{F})$ is finer than $T_{20}(\mathcal{F})$.

In $(\mathcal{F}, T_{12}(\mathcal{F}))$, $F_\alpha \to F \Longleftrightarrow$
$F = \lim_D F_\alpha = \underline{\lim}_D F_\alpha \Rightarrow \overline{\lim}_D F_\alpha \subset F \Rightarrow$
$F_\alpha \to F$,so $T_{12}(\mathcal{F})$ is finer than $T_{30}(\mathcal{F})$.

The rest is obvious.

THEOREM 3.3 Let X be a compact H ausdorff space. Then, in the collection $\mathcal{G}$ consisting of all open subsets of X, the relation between the sixteen kinds of hypertopologies indicated in figue 3. There, the topologies upper are finer than lower, and if $T_{11}(\mathcal{G})$ is compact, so is $T_{22}(\mathcal{G})$.

THEOREM 3.4 Let X be a compact Hausdorffspace. Then, in the collection $\mathcal{H}$ consisting of all clopen subsets of X, the relation between the sixteen kinds of hypertopologies indicated in Figue 4.

There, the topologies upper are finer than the lower, and two topologies at the same level line (dotted line) are homeomorphic in the following map:

$$\mathcal{H} \longrightarrow \mathcal{H}$$

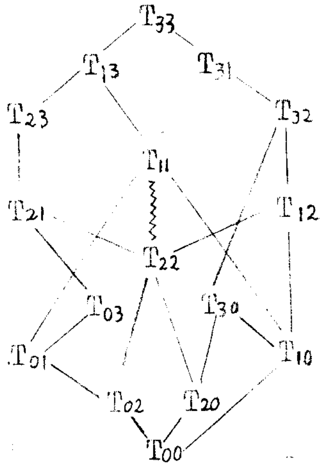
$$H \longmapsto H^c,$$

and two topologies at the same line ([illegible]) are homeomorphic in the following map:

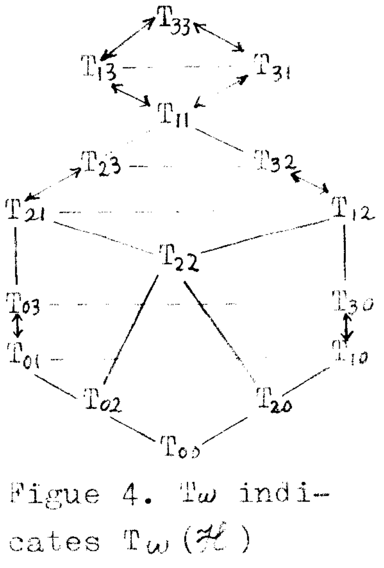
[illegible]

$H \longmapsto H$ [illegible]

COROLLARY. In the collection $\mathcal{H}$, any two topologies of T_{11}, T_{33}, T_{13}, T_{31} are homeomorphic; any two topologies of T_{23}, T_{32}, T_{21}, T_{12} are homeomorphic; any two topologies of T_{03}, T_{30}, T_{01}, T_{10} are homeomorphic; any two topologies of T_{02}, T_{20} are homeomorphic. i.e., the sixteen kinds of hypertopologies were classified [illegible] equivalent class at homeomorphic equivalent relation.



Figue 3. T_ω indicates $T_\omega(\mathcal{G})$



Figue 4. T_ω indicates $T_\omega(\mathcal{H})$

[illegible] denotes the equivalent class of T_{11}, T_{33}, T_{13}, T_{31} .

[illegible] denotes the equivalent class of T_{23}, T_{32}, T_{21}, T_{12} .

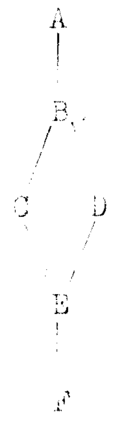
[illegible] denotes the equivalent class of T_{03}, T_{30}, T_{01}, T_{10} . .

D denotes the equivalent class of T_{22} ,T .

E denotes the equivalent class of T_{02} ,T_{20} .

F denotes $\{T_{00}\}$.

Then A,B,C,D,E,F, forms a distributive lattice under the ordering of coarseness.



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