Review of

Aspects of Vagueness

Edited by

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D. Reidel, Dordrecht, 1984.

This book gathers a collection of thirteen papers devoted to various approaches to the modeling of vagueness. About half of them use the idea of a fuzzy set as a basic concept while the others reflect different proposals. It is not possible to report on all the issues discussed in these articles within the scope of a book review, however let us briefly mention the topic of each of them in order to account for the great variety of this book. In a short introduction J. Bečvář provides a general discussion of the issue of vagueness in relation to our perception of the complex entities of the real world and their modeling by means of mathematical entities. S. Gottwald gives an overview of a neglected collection of papers written in German by Klaua and his group, who developed a 'many-valued set theory' with generalized membership functions between 1965 and 1970 ; this impressive work, which is close to fuzzy set theory but more mathematically-oriented, was done independently of Zadeh's proposal and without being motivated by any particular applications. U. Höhle and E.P. Klement propose a general mathematical setting for plausibility measures in Shafer's sense, which includes fuzzy probability measures and possibility measures as special cases, and consider entropies in this framework. In a behavioral sciences context, M. Katz presents axiomatic theories of proximity (or similarity) and dominance expressed in a first-order language with two-valued or multi-valued models where truth-values are viewed as degrees of error. D. Miller gives two papers ; the first one exhibits difficulties encountered in philosophy of sciences when the existence of degrees of truth is assessed ; the second one deals with a theory of partial identity for the elements of a Boolean algebra and establishes the equivalence of this theory with the theory of metric Boolean algebras. In the general context of social choice, S. Ovchinnikov focuses his attention on max-product transitive fuzzy relations and their representations. A. Pultr works with a fuzzy equivalence relation and views the membership degree of an element in a fuzzy set as the nearness degree of the element with the crisp subset of elements which undisputedly belong to the fuzzy set. The paper by H.J. Skala accounts for various

levels of ambiguity which are encountered when we look for mathematical representations of actual world phenomena ; more particularly the relation between vagueness and infinity is discussed in the context of non-atomic games. In a long paper A. Sochor presents the alternative set theory (or semi-sets theory) developed by P. Vopěnka [7] and his group ; basic philosophical assumptions are investigated in connection with the differences between Bolzano and Cantor approaches ; basic ideas as well as a sampling of results are nicely introduced. S. Termini provides us with an epistemological discussion of Katz' metrical approach to vagueness, Pultr's proposal and Parikh's [5] analysis of vague predicates. E. Trillas and L. Valverde propose a mathematical framework based upon *-transitive, symmetrical, "weakly" reflexive fuzzy relations for formalizing the idea of indistinguishability, where * is the operation equipping an ordered semi-group and use their indistinguishability operators for estimating degrees of fuzziness, based on the lack of distinction between a fuzzy set and its complement (see Yager [8]). The book concludes with a paper by Zadeh presenting his new theory of commonsense reasoning which deals with dispositions ; a disposition is a proposition with an implicit or explicit fuzzy quantifier ; representation issues in terms of fuzzy numbers and various pattern of reasoning aiming at chaining dispositions are discussed ; the topic of this paper is directly relevant for the treatment of certainty factors in expert systems. The papers of this book explore the various facets of vagueness and more particularly, using the nomenclature proposed by Termini, deal with a) the presence of bordeline cases, b) the existence of dispersive prime propositions (expressing the results of elementary experiments), c) the (in)coherence of natural languages, d) the nature of mathematical truth (with respect to vague predicates), e) the problem discrete vs continuum, f) the relationship between mathematics, empirical observations and reality. The relative diversity of the mathematical frameworks which are used suggests to study the existence of possible connections between them ; for instance there are similarities between Katz' proposal and fuzzy equivalence / fuzzy ordering relation-based approaches ; let us also mention Novak's work [4] about the link between semi-sets and fuzzy sets. The indiscernibility problem, discussed by Poincaré (the non-transitivity of a relation of indistinguishability or of approximate equality in physical continua) is considered in most of the papers. This question was already at the origin of the study of max-product transitive fuzzy relations by Menger [3] in probabilistic microgeometry, more than ten years before the

advent of fuzzy set theory. Recently this idea of indistinguishability (which as the belonging relation, may be a matter of degree) has been investigated by several researchers in the framework of fuzzy set theory. Apart from the papers of Pultr and of Trillas and Valverde which are included in this book, let us particularly mention more recent works by Ponasse [6] and Cerruti and Höhle [1] along this line. Another basic question with respect to vagueness is the existence (and the meaning) of intermediary truth values between 'true' and 'false'. However this problem must not be confused with the situation when there is only two truth-values ('true' and 'false') and we have different kinds of measures of confidence (probability, plausibility and credibility in the sense of Shafer, possibility in the sense of Zadeh together with the dual idea of necessity) which can be used for estimating our confidence in the truth (or the falsity) of a proposition. In the particular case of vague predicates which are evaluated with respect to a precise body of information, intermediary truth-values are encountered, see Dubois Prade [2] for a discussion. Besides these confidence measures are themselves liable to be interval-valued or even fuzzy-valued.

This book is an extremely valuable source of information for anybody who is interested in mathematical or philosophical issues related to vagueness and uncertainty. For the people of the fuzzy set community, it will help them get acquainted with alternative approaches which have been developed independently.

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