PANSYSTEMS THEORY AND ITS APPLICATION TO FUZZY INFORMATION AND NETWORK ANALYSIS

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ABSTRACT

Within the transdisciplinary framework of pansystems theory, the present paper develops some new investigations of fuzzy information and network analysis.

KEYWORDS

Pansymmetry, pansystems relations, pansystems fuzzy clustering, implicit pansystems theorems, combination-clearness-increasing principle, panoptimal selection, panweighted network.

INTRODUCTION

Pansystems theory, presented first in 1976 in China, is a newborn transdisciplinary investigation and applications of generalized system-transformation-symmetry in things mechanism. Its key points are the investigation and applications of pansymmetry, including SPIC (the relative conservations and closedness (RC) of structure-predicate-information) and the mathematical analysis of transformations of the so-called pansystems relations (PR) (which include following generalized interrelations: macromicroscopy, motion-rest, whole-parts, body-shadow, causality, observocontrol, series-parallel, simulation, shengke (synergy-opposition), panorder, clustering-discoupling, difference-identity).

Within the framework of pansystems theory, Chinese scientists obtained near one thousand new mathematical propositions and theorems, and concretely developed some new investigations concerning many disciplines such as fuzzy sets and systems, panweighted network analysis, cybernetics, large scale systems, dynamic programming, dynamic games theory, artificial intelligence, pattern recognition, theoretic biology, metaecology, metamedicine and metadiagnostics, economics, clustering analysis, universal algebra, topology, hypercomplex functions, discrete mathematics and combinatorial mathematics, approximation-transforming theory, mechanics, automata, computer, communication, science of sciences, geography, catastrophe theory, padagogy, linguistics, psychology, FTA, etc. After introducing the main outline of pansystems theory, in this paper we shift our center to develop the pansystems investigation of fuzzy information and generalized network analysis. Related concepts include: combination-clearness-increasing of fuzzy information, emergence of pansystems fuzzy clustering, CPIS (causality panproduct implicit simulation) principle in fuzzy intelligence-like control, pansystems CD (connectedness-discoupling) analysis, decomposition of large scale network, panoptimal selection, associativity principle of panweighted network, etc.

Let G be a given set, $f, g \in P(G^2)$, define $f^{-1}=\{(x,y)\mid (y,x)\in f\}$, $f\circ g=\{(x,y)\mid \exists t\in G, (x,t)\in f, (t,y)\in g\}$, $g^{(2)}=g\circ g$, $g^{(n+1)}=g^{(n)}\circ g$, $I=I(G)=\{(x,x)\mid x\in G\}$, $R[G]=\{g\mid I\leq g\}$, $S[G]=\{g\mid g=g^{-1}\}$, $T[G]=\{g\mid g^{(2)}\leq g\}$, $g^t=g\vee g^{(2)}\vee\cdots$, $E_s[G]=R[G]\cap S[G]$, $E[G]=R[G]\cap S[G]\cap T[G]$. For any given set D, the element of $E_s[G]\uparrow D$ is called <u>pansystems operators (PO)</u>, for example, $\varepsilon_1(g)=g\vee g^{-1}\vee I$, $\varepsilon_2(g)=\varepsilon_1(g\wedge g^{-1})$, $\varepsilon_3(g)=\varepsilon_1(g^t\wedge g^{-t})$, $\varepsilon_{i+5}(g)=\varepsilon_i(\bar{g})$ $(\bar{g}=G^2-g)$, $\delta_j(g)=[\varepsilon_j(g)]^t$, $\delta_0(g)=\max\{r\mid r\in E[G],\ r\leq g\}$ (provided it makes sense), $\delta_{11}(g)=\delta_0(\varepsilon_2(g))$, $\delta_{12}(g)=\delta_{11}(\bar{g})$. It is proved that $\varepsilon_i(g)\in E_s[G]$, $\delta_j(g)\in E[G]$.

<u>Basic Operations (BO)</u>: Pr (Projection), Ct (Confinement), In (Inversion: g^{-1}), Cn (Composition: $f*g$).

<u>Induced Operations (IO)</u>: Em (Embodiment, Pr^{-1}), Ex (Extension, C_t^{-1}), Hs (Hard Simulation, $P_r * E_m$), Is (Implicit Simulation, $E_m * P_r$), Ss (Synergy Simulation, $E_x * I_s * C_t$), Mi (Microscopy, $C_t * E_m$), Ma (Macroscopy, $E_x * P_r$), etc.

<u>Pansystems Operations (POn)</u> $\supset$ BO, IO, PO, Set Operations, Logic Operations, etc. $U: A \xmapsto{P} B$ means A is transformed into B by using certain composition U of POn. Sometimes "U:" will be omitted. Briefly speaking, POn = (B,I,P,S,L)O.

If $g\subset G\times G'$, then $g\circ g^{-1}\in E_s[g\circ G')$, $g^{-1}\circ g\in E_s[G\circ g)$, where $g\circ G'=\{x\mid (x,y)\in g, y\in G'\}$, etc. If Ct and Pr are defined in the sense of set theory, and g is of Hs, then we have $g\circ g^{-1}\in E[G)$, $g^{-1}\circ g\in E[G')$.

<u>RC of Is</u>: Series-Parallel, Identity-Seeking, Connectedness, Inversion, Some PO, etc.

<u>RC of Em</u> $\supset$ RC of Is, Difference-Seeking, Equivalence, Complement, Discoupling, etc.

<u>RC of Es Relations:</u> Ct, Pr, Em, Hs, Is, disjunction, conjuntion, In, Cn (for commutative case), etc.

<u>Es-Correspondence Principle</u>: For $\delta\in E_s[G)$, define $G=\cup G_i(d\delta)$, where $G_i=\max\{Q\mid Q\subset G,\ Q^2\leq\delta\}$, and denote $G/\delta=\{G_i\}$, $f_\delta=\{(x,G_i)\mid x\in G_i\}\subset G\times G/\delta$, it is an implicit simulation between G and G/δ, and $f_\delta\circ f_\delta^{-1}=\delta$. Sometimes we adopt notation $G_i\subset G(d\delta)$. If $g\subset G\times G'$ is an implicit simulation or g is covering on G and G', then $g\circ g^{-1}\in E_s[G)$, $g^{-1}\circ g\in E_s[G')$. If $\delta\in E[G)$, then $G=\cup G_i(d\delta)$ is reduced to an exact partition.

<u>Theorem 1</u>. Let $Q\subset P(G^2)$. If Q forms a group (for Cn, In and $I(G)$), then $\delta=\vee f(f\in Q)\in E[G)$; if Q is a semigroup, then $\delta\in T[G)$; if Q is a monoid, then $\delta\in R\cap T[G)$; if $I\in Q$, and $f\in Q$ implies $f^{-1}\in Q$, then $\delta\in E_s[G)$. Furthermore, for the first and fourth cases mentioned, we have $\delta=\vee f^{-1}=\vee f\circ f^{-1}=\vee f^{-1}\circ f$.

<u>CD-Decomposition Analysis</u>: If $\delta=\varepsilon_6(\delta')$, $\delta'\in E_s[G)$, then $\delta'=\varepsilon_6(\delta)$. If $g\in P(G^2)$, then $\delta'=\varepsilon_7(g)$, $\varepsilon_6(g)$, $\varepsilon_6(g^t)$ for $\delta=\varepsilon_1(g)$, $\varepsilon_2(g)$, $\varepsilon_3(g)$ respectively. If $G=\cup G_i(d\,\delta(f))$, δ is some PO (for example $\delta=\varepsilon_k, \delta_j$), $f=g^{(n)}$, g^t, $(I\vee g)^{(n)}$, then in G_i and in $G/\delta(f)$ there is some connectedness relation and discoupling relation respectively for $k,j\leq 5$. Specially, in $G_i\subset G(d\delta_1(g))$ there is of E-type weak connectedness, and in $G_i\subset G(d\delta_3(g))$ the strong connectedness. Generally speaking, the correspondences $(G_i, G/\delta(f))$, $(\delta, \varepsilon_6(\delta))$ realize the transformations between CD-relation. Furthermore, $\delta(f)\neq G^2$ is the necessary and sufficient condition of G possessing related decomposition whose blocks G_i are of CD-property of $\delta(f)$-type. For example, if $\delta_1(g)\neq G^2$, then G can be partitioned into $G/\delta_1(g)$ which is without the g-connectedness. If $\delta_3(g)\neq G^2$, then among G_i we at most find unidirectional g-series connectedness, and in G_i any two points are of strong connectedness.

CLEARNESS-INCREASING OF UNCERTAIN RELATIONS AND FUZZY INFORMATION

Generally speaking, many uncertain relations and fuzzy information are described by general binary relations which can be reduced to the implicit simulations, hard simulations or body-shadow relations, and the RC-principles mentioned in above section present a sort of determinedness or clearness in the degree of certain relations. The reduction of general pansystems simulation to a body-shadow relation is called pansystems emergence which plays certain roles of clearness--increasing. Another emergence is concerning the reduction of $E_s[G)$ to $E[G)$ which increases the clearness of clustering. Naturally, these sorts of emergences are closely connected with each other and pansystems theory has presented concrete transformations.

Emergence Theorem of Is. If $g \subset G \times G'$ is an implicit simulation, then $\varphi = (g \cdot g^{-1})^{(r)} \circ g \subset G \times G'$ is also an implicit simulation and $\varphi \circ \varphi^{-1} = (g \circ g^{-1})^{(2r+1)}$. Furthermore, $f = (g \circ g^{-1})^{t} \circ g \subset G \times G'$ is a hard simulation and $f \circ f^{-1} = (g \circ g^{-1})^{t}$.

Emergence Theorem of Es. If $\delta \in E_s[G)$, then $\delta^{(m)} \leq \delta^{(m+n)}$, and $\delta^{(\infty)} = \delta^{t} \in E[G)$.

Implicit Pansystems Theorem. If $g \subset G \times G'$ is an implicit simulation, $\delta' \in S[G')$, $\delta \in E_s[G)$, $\delta \leq \mathcal{E}_7(g \circ \delta' \circ g^{-1})$, $G_i \subset G\ (d\delta)$, then on $G_i \times G_i \circ g$, g is reduced to an embodiment.

Combination-Clearness-Increasing Theorem. Let $g(\sigma) \subset F(\sigma) \times G(\sigma)$ be some implicit simulations, $\sigma \in \Sigma$, and $F = \Pi F(\sigma)$, $G = \Pi G(\sigma)$, $g = \varphi(\Pi g(\sigma))$, $\theta(\sigma) \in S[G(\sigma))$, where $\varphi : \Pi(F(\sigma) \times G(\sigma)) \longrightarrow F \times G$ be 1-1 mapping by using the transformation of order of coordinate axes. If $Q(\sigma)^2 \leq \mathcal{E}_7(g(\sigma) \circ \theta(\sigma) \circ g(\sigma)^{-1})$, then $(\Pi Q(\sigma))^2 \leq \mathcal{E}_7(g \circ \theta \circ g^{-1})$, where $\theta = \Pi \theta(\sigma)$.

According to the above two theorems, $g(\sigma)$ and g are of clear communication on $Q(\sigma) \times Q(\sigma) \circ g(\sigma)$ and on $(\Pi Q(\sigma)) \times (\Pi Q(\sigma)) \circ g$ respectively, and the latter plays a role of combination clearness--increasing with respect to the former.

PANSYSTEMS OBSERVOCONTROLLABILITY AND PANOPTIMAL DESIGN OF DIAGNOSTICABILITY OF NETWORK

Let $F(\sigma, \lambda)$ be certain sets with parameters σ and λ, $F = \{F(\sigma, \lambda)\}$, $J_\lambda \subset \Pi F(\sigma, \lambda)$, $J = \{J_\lambda\}$, then $S = (F, J)$ can be considered as a primitive model of things-describing. Let $S \overset{p}{\longmapsto} f_i, g_j$, where $f_i \subset U_i \times V_i \times G_i^2 \in F^4$, $g_j \subset U_j \times V_j \times G_j \times H_j \in F^4$, and $U_i\,(U_j)$, $V_i\,(V_j)$ represent synergy and opposition parameter spaces respectively. Let $\tau = \{T_k\}$, $Q^* = \cup Q \uparrow T_k$, and $\{f_i\} \cup \{g_j\} \overset{p}{\longmapsto} \varphi_m \subset E_m \times H^2$, $\psi_n \subset E_n \times H$, where $E_m, E_n, H \in \{Q \uparrow T_k, Q^*\}$, and φ_m is implicit simulation between E and H^2, and ψ_n is implicit simulation between E and H, $Q \in F$.

Theorem of Pansystems Forcing Block Observocontrollability (Fboc). Under the above description, system S contains the Fboc $H^2 \simeq \cup R_r(d \vee \varphi_m^{-1} \circ \varphi_m)$; $H / \wedge \psi_n^{-1} \circ \psi_n$.

Based the Fboc theorem and implicit pansystems theorem, we can obtain

Theorem 2. Let $\varepsilon \in S[E)$, $\delta \leq \mathcal{E}_7(\varphi_m^{-1} \circ \varepsilon \circ \varphi_m)$, $R \subset H^2(d\delta)$, then $\varphi_m \circ R\ (\subset E_m)$ can control the transformation R. If $\delta' \leq \mathcal{E}_7(\psi_n^{-1} \circ \varepsilon \circ \psi_n)$, $R' \subset H(d\delta')$, then R' can be observed from $\psi_n \circ R'$.

As distinct from Fboc theorem, the theorem 2 can be considered as a partial observocontrollability (Poc) theorem. They all are connected with the clearness-increasing of uncertain relations (including certain uncertain automata and fuzzy information). If H is the basic state space, and the practical requirement of diagnosis of reliability is described as $\delta \in E_s[H)$, then to select $\{(E_n, \psi_n)\}$ such that $\wedge \psi_n^{-1} \circ \psi_n \leq \delta$ is called panoptimal selection. Under a given condition, the design according to panoptimal selection principle is called panoptimal design of diagnosticability. For the case of digital network, generally speaking, the diagnostic terminals are usually confined and $E = \Pi E_n$ may be described as $\Pi(B_i \uparrow r(n))$, consequently, the key point of design of reliability diagnosis is reduced to search some reasonable connections ψ_n. Naturally, the concept of H may be of very wide sense, its element sometimes can be described as certain processes. Hence the pansystems model of diagnosticability design is quite universal, and can be used to, for example, the reliability study of software network, computer communication network, operating system and so-called process-network, naturally including certain problems of power network and social diagnostics.

Pansystems Quantization: The concepts of quantity and form in mathematics are always developing. Substantially, any generalized system or panstructure may be considered as a sort of quantity, provided it possesses abundant concrete properties (specially, operationable properties). Generalized quantity system can be constructed by using synergy simulation step by step from the traditional quantity systems, for example: R^n, N, interval number system, hypercomplex number systems, $^*R$, fuzzy number systems, group, ring, field, modulus, lattice, tensor, etc. Pansystems methodology, in fact, presented the $E_s[G)$, some panorder structures, panweighted networks and various pansystems simulations as certain new primitive quantity systems. The pansystems model (specially, the implicit model or hard model) of things-object built by generalized quantity is called pansystems quantization.

Principle of Causality-Panproduct Implicit Simulation (CPIS): The causes (observations) and effects (control) of things-process and their modifiers all can be represented by using direct products of some abstract sets or their pansystems quantization, and the causality or observocontrol relations usually are described as an implicit simulation of these products or their pansystems quantizations. By using this concept one can treat or model many intelligence-like control processes, including some fuzzy control processes.

PANSYSTEMS LOGIC OF PANWEIGHTED NETWORK

The investigation of whole-parts relations and body-shadow relations and its applications to methodology and reasoning is called pansystems logic, its key points are still related SPIC and mathematical transformations of PR. The investigation of pansystems logic of panweighted network $g \in W \uparrow G^2$ or $g \subset G^2 \times W$ presents some new developments and unification of dynamic programming, network analysis, dynamic games theory, etc.

Yinyang-Reducing Principle: Let $g \subset G^2 \times W$ be a panweighted network, $R \subset W$, then $g \circ R \subset G^2$. By using the pansystems CD-analysis, we can explain the block relations of $G = \cup G_i (d\delta(f))$ in terms of the panweight W, where $f \in \{(g \circ R)^{(n)}, (g \circ R)^t, (I \vee g \circ R)^{(m)}\}$, δ is certain PO. For example, $\delta(f) = \mathcal{E}_1((g \circ R)^t)$, if $x, y \in G_i$, $x \neq y$, then there exists a positive integer n and $\omega_i \in R$, $i = 1, 2, \ldots, n$, such that $(x, y) \in (\circ)\text{-}\Pi(g \circ \omega_i)$ (composition product), or $(x, y) \in (\circ)\text{-}\Pi(g \circ \omega_i)^{-1}$. And the former is explained as that we can connect x and y by paths which possess panweights $\omega_1, \omega_2, \ldots, \omega_n$ serially.

RC of Associativity: Let $\theta_1 : W \uparrow G \longrightarrow W$, $\theta_2 : W^2 \longrightarrow W$, $\theta = (\theta_1, \theta_2) : (W \uparrow G^2)^2 \longrightarrow W \uparrow G^2$, $(g \theta f)(x, y) = \theta_1 \{ g(x,t) \theta_2 f(t,y) \mid t \in G \}$. If θ_1, θ_2 possess associativity and distrbutivity, then θ is also of associativity. Here θ_1 describes a sort of selection called panoptimal selection, and induces $\bar{\theta}_1 : (W \uparrow G^2) \uparrow G \rightarrow W \uparrow G^2$, similarly, we have induced $\bar{\theta}_2 : (W \uparrow G^2)^2 \longrightarrow W \uparrow G^2$. If θ_i is distributive with respect to θ_2, so does θ to $\bar{\theta}_i$. If θ_2 is commutative, $g \bar{\theta}_i g = g$, then $(g \theta f)^{-1} = f^{-1} \theta g^{-1}$, $(e \bar{\theta}_i g)^{(n)} = e \bar{\theta}_i g \bar{\theta}_i \cdots \bar{\theta}_i g^{(n)}$, where $g^{(n)} = g_1 \theta g_2 \theta \cdots \theta g_n (g_k = g)$, $g \theta e = e \theta g = g$. Pansystems network analysis unifies to treat both fuzzy and non-fuzzy cases.

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