134

Logical Analysis for a Class of Complex Systems

Zhang Nanlun Huang Shuqing**

Wuhan Institute of Building Materials

This paper proposes a logical method of analysing complex systems. Through seeking the logical connections between data we can study the relation between the complex system variables.Analysing some examples shows that this analysis is very analogous to the thought of the human's brain. It is possible to provide a way to study the complex systems.

In order to solve the method of logical analysis for complex systems this paper introduces a concept to a class of complex switches in the first section, recounts the simplifying method by the Pan-Karnaugh diagram in the section, and recommends some practical examples of logical analysis of system in the third section.

1. Concept of A Class of Complex Switches

If the switch can only output an information (1 or 0) in each action (switch on or switch off) then the switch is called a simple switch.

If the switch can output more than two information in each action at the same time and these information can control more than two circuits which are independent of each other, then the switch is said a complex switch.

In this paper we only discuss a class of complex switches which is shown in figure. 1.

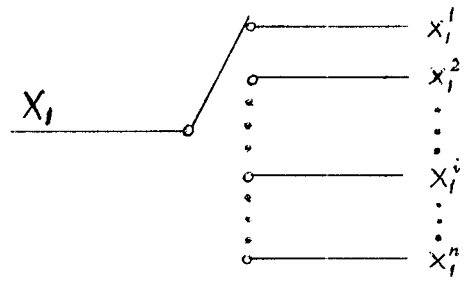


Fig. 1. the construction figure of complex switch X_1 having n branches

** He is now a visiting scholar at Washington University in St. Louis, MO 63130, U.S.A.

The above switch can be shown in vector form as

$X=(x_1^1, x_1^2, \ldots, x_1^i, \ldots, x_1^n)$

where the upper index of letter indicates the branch number of switch. The set of all branch number is $A=\{1,2,\ldots,i,\ldots,n\}$. The lower index of letter indicates the switch itself.

Definition 1: the internal AND,OR,NOT operations for complex switches satisfy the following law in tables (take X_1 for example).

AND	x_1^1	x_1^2	$\cdots$	x_1^i	$\cdots$	x_1^n
x_1^1	x_1^1	0	$\cdots$	0	$\cdots$	0
x_1^2	0	x_1^2	$\cdots$	0	$\cdots$	0
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_1^i	0	0	$\cdots$	x_1^i	$\cdots$	0
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_1^n	0	0	$\cdots$	0	$\cdots$	x_1^n

(1)

OR	x_1^1	x_1^2	$\cdots$	x_1^i	$\cdots$	x_1^n
x_1^1	x_1^1	$x_1^1+x_1^2$	$\cdots$	$x_1^1+x_1^i$	$\cdots$	$x_1^1+x_1^n$
x_1^2	$x_1^2+x_1^1$	x_1^2	$\cdots$	$x_1^2+x_1^i$	$\cdots$	$x_1^2+x_1^n$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_1^i	$x_1^i+x_1^1$	$x_1^i+x_1^2$	$\cdots$	x_1^i	$\cdots$	$x_1^i+x_1^n$
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_1^n	$x_1^n+x_1^1$	$x_1^n+x_1^2$	$\cdots$	$x_1^n+x_1^i$	$\cdots$	x_1^n

(2)

NOT	
x_1^1	$x_1^2+\cdots+x_1^i+\cdots+x_1^n=\sum x_1^i \quad (i \neq 1)$
x_1^2	$x_1^1+x_1^3+\cdots+x_1^i+\cdots+x_1^n=\sum x_1^i \quad (i \neq 2)$
$\vdots$	$\cdots$
x_1^i	$x_1^1+x_1^2+\cdots+x_1^{i-1}+x_1^{i+1}+\cdots+x_1^n=\sum x_1^i \quad (i \neq i)$
$\vdots$	$\cdots$
x_1^n	$x_1^1+x_1^2+\cdots+x_1^i+\cdots+x_1^{n-1}=\sum x_1^i \quad (i \neq n)$

(3)

Introduction of the following concept is of benefit: '1' indicates $x_1^1, x_1^2, \ldots, x_1^i, \ldots, x_1^n$ are fully connected, '0' fully broken off, x_1^i that is the ith branch of switch X_1 is connected, $\overline{x}_1^i$ that is the ith branch of switch X_1 is broken off.

Obviously

$$X_1^1+X_1^2+\ldots+X_1^i+\ldots+X_1^n=X_1^1+\sum_{i\neq 1}X_1^i=X_1^1+\overline{X}_1^1=1 \qquad (4)$$

The formula just shows the construction of switch X_1 in Fig.1.

External operations and characters of complex switches are the same as that of switch in the normal switch algebra (omited).

The table is arranged to compare with the internal AND, OR, NOT operations of simple and complex switch in table 1.

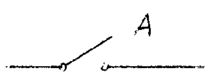
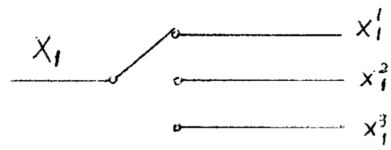
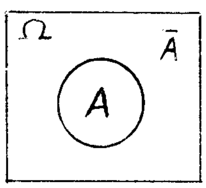
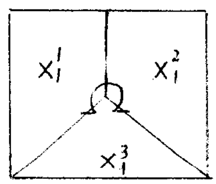
	simple switch		complex switch (suppose the branch number=3)	
construction				
Venn diagram				
operational rule	$A+\overline{A}=1$	(5)	$x_1^1+\overline{x}_1^1=x_1^1+(x_1^2+x_1^3)=1$	(5')
	$A\cdot\overline{A}=0$	(6)	$x_1^1\cdot(x_1^2+x_1^3)=0$	(6')
	$A+A=A$	(7)	$x_1^1+x_1^1=x_1^1$	(7')
	$A\ A=A$	(8)	$x_1^1\cdot x_1^1=x_1^1$	(8')
	$A+1=1$	(9)	$x_1^1+1=1$	(9')
	$A\cdot 1=A$	(10)	$x_1^1\cdot 1=x$	(10')
	$A+0=A$	(11)	$x_1^1+0=x$	(11')
	$A\cdot 0=0$	(12)	$x_1^1\cdot 0=0$	(12')
	$\overline{\overline{A}}=A$	(13)	$\overline{\overline{x}}_1^1=\overline{x_1^2+x_1^3}=x_1^1$	(13')

Table 1. operational rule table

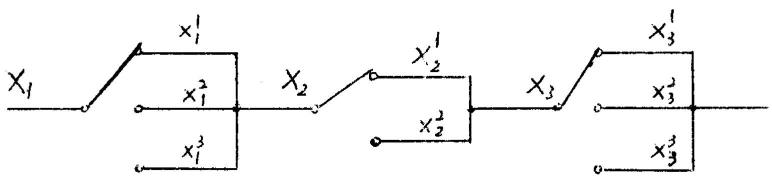
So-called switch variable is the same thing as switch. The switch variable together with the compound switch combined by switch variable through limit time operations (AND, OR, NOT) are refered to as switch function. To understand the action of switch function, we should define the Pan-minterm first.

Definition 2. So-called a Pan-minterm of switch function is a AND term including all variables in function. Each variable appears once and only once as a factor in its original form, i.e.

$$X^A = X_1^{A_1} X_2^{A_2} \ldots X_i^{A_i} \ldots X_n^{A_n}$$

where $X_1, X_2, \ldots X_i \ldots X_n$ are switch variables, and $A_1, A_2, \ldots, A_i, \ldots, A_n$ are a set of branch number corresponding to switch function. X^A is called a Pan-minterm for $X_1, X_2, \ldots, X_i, \ldots, X_n$.

Example 1. write down the Pan-minterm of compound switch shown in the following diagram.



The Pan-minterm for X_1, X_2, X_3 is:

$x_1^1x_2^1x_3^1, x_1^1x_2^1x_3^2, x_1^1x_2^1x_3^3, x_1^1x_2^2x_3^1, x_1^1x_2^2x_3^2, x_1^1x_2^2x_3^3, x_1^2x_2^1x_3^1, x_1^2x_2^1x_3^2, x_1^2x_2^1x_3^3, x_1^2x_2^2x_3^1, x_1^2x_2^2x_3^2, x_1^2x_2^2x_3^3, x_1^3x_2^1x_3^1, x_1^3x_2^1x_3^2, x_1^3x_2^1x_3^3, x_1^3x_2^2x_3^1, x_1^3x_2^2x_3^2, x_1^3x_2^2x_3^3$

Theorem 1: Suppose $p_1, \ldots, p_n$ are the numbers of corresponding element in the set of number of variable branch, then the number of Pan-minterm on $X_1, \ldots, X_n$ will be $\prod_n p_i$, $\prod_n p_i = p_1 \times \ldots \times p_n$.

Pan-minterm prossesses the following characteristic:

Characteristic 1. If the sum of all the Pan-minterm for $X_1, \ldots, X_n$ is labeled as $U_A X^A$, then $U_A X^A = 1$

Characteristic 2. If the product of all the Pan-minterm for $X_1, \ldots, X_n$ is iabeled as $\prod_A X^A$, then $\prod_A X^A = 0$.

Theorem 2: any switch function $f(X_1^{A_1}, \ldots X_n^{A_n})$ can be expressed by the sum of certain Pan-minterm for $X_1, \ldots, X_n$, and this expansion is the sole one.

2. Pan-Karnaugh Diagram

Pan-Karnaugh Diagram is a spread of normal Karnaugh Diagram. Now we take an example to illustrate how to draw the Pan-Karnaugh Diagram.

Example 2: Suppose $X^A = X_1^{A_1} X_2^{A_2} X_3^{A_3} X_4^{A_4} X_5^{A_5}$

where $A_1=\{1,2,3\}$, $A_2=\{1,2\}$, $A_3=\{1,2\}$, $A_4=\{1,2,3,4\}$, $A_5=\{1,2\}$ The Pan-karnaugh Diagram of above example has the following form.

X_5	X_3	X_1 \ X_4, X_2	X_4^1		X_4^2		X_4^3		X_4^4	
			X_2^1	X_2^2	X_2^1	X_2^2	X_2^1	X_2^2	X_2^1	X_2^2
X_5^1	X_3^1	X_1^1	11111 12345	12111 12345	11121 12345	12121 12345	11131 12345	12131 12345	11141 12345	12141 12345
		X_1^2	21111 12345	22111 12345	21121 12345	22121 12345	21131 12345	22131 12345	21141 12345	22141 12345
		X_1^3	31111 12345	32111 12345	31121 12345	32121 12345	31131 12345	32131 12345	31141 12345	32141 12345
	X_3^2	X_1^1	11211 12345	12211 12345	11221 12345	12221 12345	11231 12345	12231 12345	11241 12345	12241 12345
		X_1^2	21211 12345	22211 12345	21221 12345	22221 12345	21231 12345	22231 12345	21241 12345	22241 12345
		X_1^3	31211 12345	32211 12345	31221 12345	32221 12345	31231 12345	32231 12345	31241 12345	32241 12345
X_5^2	X_3^1	X_1^1	11112 12345	12112 12345	11122 12345	12122 12345	11132 12345	12132 12345	11142 12345	12142 12345
		X_1^2	21112 12345	22112 12345	21122 12345	22122 12345	21132 12345	22132 12345	21142 12345	22142 12345
		X_1^3	31112 12345	32112 12345	31122 12345	32122 12345	31132 12345	32132 12345	31142 12345	32142 12345
	X_3^2	X_1^1	11212 12345	12212 12345	11222 12345	12222 12345	11232 12345	12232 12345	11242 12345	12242 12345
		X_1^2	21212 12345	22212 12345	21222 12345	22222 12345	21232 12345	22232 12345	21242 12345	22242 12345
		X_1^3	31212 12345	32212 12345	31222 12345	32222 12345	31232 12345	32232 12345	31242 12345	32242 12345

Table 2.

The numbers in small square of diagram indicate the Pan-minterm. For example [12121 / 12345] shows the square is of a Pan-minterm $x_1^1 x_2^2 x_3^1 x_4^2 x_5^1$

The construction characteristics in the above table are: the various variables are arranged from internal to external, X_1, X_2 are in the most internal, then X_3, X_4 and then X_5. The variable values of X_1 ,X_3,X_5,are ranged vertically, and that of X_2 ,X_4 horizontally. If certain numble of variables had to be increased, only new square is added, while the position of original variable is still remain unchanged.

The corresponding terms of squares can be merged or not that is determined by whether these terms are logically adjacent in the Pan-Karnaugh diagram.

Definition 3: Logical adjacency means that:

1. For more then two squares within one horizontal row (or vertical column), if the longest line between these squares is within internal layer variables X_1 or X_2 , and the number of squares is equal to the number of variable branch of X_1 or X_2 , then these squares are said to be logical adjacency at X_1 or X_2 .

2. For more than two squares within one horizontal row (or vertical column), if the longest line between the squares is within external layer variables X_i and the number of squares is equal to the number of variable branchs of X_i and also the squares possess of the same position at one side of all the longest line (including the boundary line of variable X_i at this moment), then these squares are said to be logical adjacency at X_i .

For example, squares $\boxed{\begin{array}{c}22221\\12345\end{array}}$ and $\boxed{\begin{array}{c}12221\\12345\end{array}}$, $\boxed{\begin{array}{c}32221\\12345\end{array}}$ is considered to be adjacent logically, because the longest line between these squares is within variable X_1 and the number of squares 3 is equal to the number of branchs 3 of variable X_1.

Another example, the squares $\boxed{\begin{array}{c}22221\\12345\end{array}}$ and $\boxed{\begin{array}{c}22211\\12345\end{array}}$ $\boxed{\begin{array}{c}22231\\12345\end{array}}$ $\boxed{\begin{array}{c}22241\\12345\end{array}}$ are considered to be logically adjacency, because the longest line between these squares possess of variable X_4 and the number of squares 4 is equal to the number of branchs of X_4 and furthermore the four squares possess the same position at one side of all the longest lines.

The third example, even though the squares $\boxed{\begin{array}{c}31231\\21345\end{array}}$ and $\boxed{\begin{array}{c}11132\\12345\end{array}}$ are adjacent in position, but not logically adjacet, because even the longest line between squares is wihtin X_5 , but two squares are not the same position of one side of the longest line.

It is obviously that the mergence of the Pan-minterm by using Pan-Karnaugh diagram is essentially equivalent to use the construction from of complex switch repeatedly (4)

$$x_1^1+x_1^2+\ldots+x_1^i+\ldots+x_1^n=1$$

By merging the Pan-minterm and eliminating surplus factor, the simplified formula can be obtained.

Theorem 3: If it is assumed that $P_1,\ldots,P_n$ are the numbers of corresponding element in the number set of variable branch $A_1,\ldots,A_n$, then the number of adjacent Pan-minterm of any Pan-minterm concerning $X_1,\ldots,X_n$ will be $\sum_{i=1}^{n} P_i - n$.

The method to simply Pan-Karnaugh diagram is described as follows:

Example 3, in terms of Pan-Karnaugh diagram, we simplify the switch function in [1].

$$Z=x_3^1x_2^1x_1^1+x_3^1x_2^5x_1^1+x_3^1x_2^1x_1^2+x_3^1x_2^2x_1^2+x_3^1x_2^3x_1^2+x_3^1x_2^4x_1^2+x_3^1x_2^5x_1^2+x_3^1x_2^1x_1^3$$
$$+x_3^1x_2^2x_1^3+x_3^1x_2^3x_1^3+x_3^1x_2^4x_1^3+x_3^1x_2^5x_1^3+x_3^2x_2^5x_1^1+x_3^2x_2^1x_1^2+x_3^2x_2^3x_1^3+x_3^2x_2^4x_1^2+x_3^2x_2^5x_1^2$$
$$+x_3^2x_2^1x_1^3+x_3^2x_2^4x_1^3+x_3^2x_2^5x_1^3+x_3^3x_2^4x_1^2+x_3^3x_2^5x_1^2+x_3^3x_2^3x_1^3+x_3^3x_2^4x_1^3+x_3^3x_2^5x_1^2$$

Solution:

Step 1, make drawing the Pan-Karnaugh diagram of function;

Step 2, enclose the adjacency term according to the above method;

Step 3, select the product-term and write down the simplified function expression.

The principles to be followed for selecting product-term are:

a. the simplified AND-OR expression must contain all the Pan-minterm in the functiion;

b. the total product-term selected should be the least;

c. the factors contained in each product-term should be the least.

Based on these principles the above function can be simplified as follows:

$$Z=x_3^1x_1^2+x_3^1x_1^3+x_3^1x_2^1+x_3^1x_2^5+x_3^2x_2^5+x_2^3x_1^3+x_2^4x_1^2+x_2^4x_1^3+x_2^5x_1^2+x_2^5x_1^3+x_3^2x_2^1x_1^2$$
$$+x_3^2x_2^1x_1^3$$

Step 4, by using the construction formula (4), we can also write down the above example in following form:

$$Z=x_3^1\overline{x_1^1}+x_3^1x_2^1+\overline{x_2^3}x_2^5+x_2^3x_1^3+x_2^4\overline{x_1^1}+x_2^5\overline{x_1^1}+x_3^2x_2^1\overline{x_1^1}$$

3. Practical Examples of Logical Analysis for A Class of Complex Systems

So far the logical analysis of variable never consider the quantitative relationship of vairable. This paper tries to combine logic with quantity. Within the frame of new logic system, we can set up a Data-Logic Method of systerm and simplify the model.

To solve the logiacl analysis of the complex systerms should pay attention to the following three aspects:

1. The relation between the variables of a complex systerm which can not use the means of determinacy is shown by a lot of data, so the statistical method should be adopted;

2. The logical method should be adopted. The so-called logical method includes the mathematical logic and the judgement of man's thought.

3. The complexity and the accuracy are a couple of contradictories. It is impossible to get the accuracy answer of complex system. Therefore, allowing the obtained answer is the approximated one (to a certain extent) can be regarded as a certain result.

The so-called logical analysis of system is shown to describe the logical functions between the variables of system studied and simplify the functions,we can thereby obtain the most simple logical expression between system variables.

The logical analysis of complex systems is carried out as following:

Step 1. the various cases of changing in variable are classed to transfer the relation between variables into the logical problem.

Step 2. write out the logical expression of relation between the variables.

Step 3. simplify the logical expressions.

Table 3.

X_3	X_1 \ X_2	x_2^1	x_2^2	x_2^3	x_2^4	x_2^5
x_3^1	x_1^1	z				z
	x_1^2	z	z	z	z	z
	x_1^3	z	z	z	z	z
x_3^2	x_1^1					z
	x_1^2	z			z	z
	x_1^3	z		z	z	z
x_3^3	x_1^1					
	x_1^2				z	z
	x_1^3			z	z	z

REFERENCE

1. Peray, K.E and Waddell. J.J., The Rotary Cement Kiln, Chemical Publishing Co. 1972.