ON ECONOMIC APPLICATIONS OF FUZZY PROGRAMMING

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1. Introduction.

Using the algebraic machinery developed in the paper [2] we presented in the paper [3] a complete theory of fuzzy programming. In this paper an economic application of fuzzy programming is presented. At first the production problem is described and then the diet problem is formulated.

2. The production problem.

Consider a production system, say a factory, in which n goods $T_1, \dots, T_n$ are involved either as inputs to the productive process or final goods. Let us assume that they have a various preferences grades.

Definition. A fuzzy activity P involving n goods corresponds to a fuzzy vector $a = (x_i, a_i)$. The good T_i is called an input to the activity if x_i is negative, and an output of the activity if x_i is a positive.
A number a_i denotes the index of a quality of i-th good which is produced (or consumed) in the x_i quantity.

An fuzzy production model involving n goods consist of a set of such activities $P_1, \dots, P_m$. Such a model is completely described by an fuzzy matrix $A = (x_{ij}, a_{ij})$, where x_{ij} is the amount of T_j produced (or consumed) when P_i is operated at unit level. The number a_{ij} denotes the index of a quality of j-th good which is produced (or consumed) by the activity P_i in the x_{ij} quantity. The fuzzy matrix A is called the production matrix of the fuzzy production model.
A production schedule of fuzzy production model is defined to be an fuzzy point with the nonnegative support. The coordinates of this fuzzy point are called intensivities for the activities P_i. If $\{a\}$ denotes a production schedule then $\mu_{\{a\}}(a)$ will denote its quality.

a) Production to meet given demand at minimum cost.

Assume we have a fuzzy production model in which is described

a minimum amount of production of T_j by an fuzzy element (y_j, b_j). The number b_j denotes a minimum quality of T_j.
Suppose further that the cost of operating the process P_i at unit level is described by fuzzy element $(z_i, 1)$.

The problem is then to choose a production schedule $\{a^*\}_P$ such that

$$\{a\}_P \cdot c \geqslant \{a^*\}_P \cdot c \ , \qquad \forall a \in R^n_+ \ ,$$

$$A \cdot \{a\} \geqslant b \ ,$$

where

$c = (z_i, 1)$, $b = (y_j, b_j)$ and we have the second kind inequalities.
Let us note that the quality of the production schedule $\{a\}_P$ is described by a numper $\mu_P(a)$ from interval $\langle 0, 1 \rangle$ (the less number the greater quality). This number desribed an "average" quality of all goods which are obtained by the all activities operating at level a .

The question of feasibility is no longer a simple one. It may easily happen that it is not technologically possible to satisfy the given demands with the given resources. If, however, a feasible schedule does not exist then the problem of optimal schedule is the problem of optimal solution of fuzzy programming (see [3]).

b) Production to maximize income from given resources.

Again we consider fuzzy production model. It is clear that the same goods production by the various activities can have different consumer's preferences. Let a_{ij} denotes the degree of consumer's

preferences of the good T_j produced in the x_{ij} quantity by the the activity P_i .
Let $(z_i, 1)$ be the rate of return or income associated with the activity P_i. However, let us assume that y_j denotes the maximum amount of production of T_j and b_j the maximum degree of consumer's preferences.

The problem is now to find a production schedule $\{a^*\}_P$ such that

$$\{a^*\}_P \cdot c \geq \{a\}_P \cdot c \quad , \qquad \forall a \in R_+^n \quad ,$$

$$A \cdot \{a\} \leq b \, ,$$

where $c = (z_i, 1)$, $b = (y_j, b_j)$.
Of course the quality of production schedule is also described by the number $\mu_P(a)$. In this case the greater quality the greater number.

3. The diet problem.

This problem has become the classical ilustration in linear programming (see for example [1]). Now, we will formulate this problem with the some conditions.

Let us assume that n dietitian is confronted with n different foods which will be labeled B_1 , ... , B_n . From these he is to select a diet, that is to be consumed annually by a person or group of persons. This yearly menu is required to supply certain amounts of various nutritional elements. We shall refer to those types of nutritive elements simply as nutrients of which there will be m varieties denoted by S_1 , ... , S_m . From biochemical analysis of foods we give the amount of the i-th nutrient contained in one unit of the j-th food. It is clear that the degree of an assimilate by the organism of the given nutrient depend on the food in which this nutrient is.

Let (x_{ij},a_{ij}) be a fuzzy element. The element x_{ij} denotes the amount of the i-th nutrient in one unit of the j-th food. The number a_{ij} denotes the degree of an assimilate of the i-th nutrient which is in the j-th food. So, the information which the dietitian needs is then conveniently presented as the fuzzy matrix $A = (x_{ij},a_{ij})$. The dietitian choose a diet. This means that he determine the amounts of B_j which each consumer shall be consumed. This diet with regard on the above degrees of an assimilate will be less or greater valuable. It will described by the fuzzy point $\{a\}$.

Let us assume that a minimum amount of S_i in the diet and minimum degree of assimilate of S_i are given. So, the nutritional requirements for S_i we may described by the fuzzy element (y_i,b_i). The condition that the diet satisfy all requirements is that the fuzzy point $\{a\}$ satisfy the following second kind inequalities

$$A\cdot\{a\} \geqslant b \qquad (*)$$

where $b = (y_i,b_i)$.

A diet for which conditions (*) are satisfied will be termed a feasible diet.

The dietitian must choose the most economical diet consistent with the requirement (*). We are assuming that a price is associated with each food. Let $c = (z_j, 1)$ denotes a fuzzy price vector. We can now give a complete statement of the diet problem. Among all diets satisfying conditions (*) find $\{a^*\}$ such that

$$\{a^*\} \cdot c \leq \{a\} \cdot c \quad , \qquad \forall\ a \in R^n_+ \qquad (**)$$

A diet which satisfies both (*) and (**) is called an optimal diet. Let us note that the diet problem is the fuzzy programming problem and if there exists the optimal solution of fuzzy programming problem then there exists the optimal diet in the diet problem.

References

[1] Gale,D, The Theory of Linear Economic Models, New York,Toronto, London 1960,

[2] Matłoka,M, On fuzzy vectors and fuzzy matrices, Busefal 19 (1984) 92-105.

[3] Matłoka,M, On fuzzy programming , Busefal (in print).