ON INDEXED FUZZY MODELS OF PRODUCTION

Part 1: The simple interval indexed fuzzy model and interval indexed fuzzy Leontief model.

Marian MATŁOKA

Institute of Economical Cybernetics, Department of Mathematics, Economic Academy of Poznań, ul. Marchlewskiego 146/150, 60-967 Poznań, Poland

1. Introduction. One of the main tasks of economical sciences is to construct models at which we first of all want to establish that they are logically possible and consistent,and next make sure that our models are realistic. The fuzzy sets theory - initiated by Zadeh (1965) provides a methodology and mathematical apparatus more adequate then the crisp ones. So,recently these new ideas are have found their expression in papers initiated a new branch of economical sciences,the fuzzy economy.Indepedently of these investigations many authors build up an interval theory of the economical systems with wealth constraints (comp.Matłoka (1981),Rohn (1978 79)). This theory is developed of basic concepts of interval mathematics.This paper is a trial of compound fuzzy and interval mathematics and application.

Because Zadeh's theory does not allow to take specific properties of economic processes into consideration,so we will use the indexed fuzzy subsets theory which was introduced by Matłoka (1984). The interval indexed fuzzy matrix theory which we will use in this paper is a generalization of fuzzy matrix theory (Matłoka (1984)).

This paper presents a preliminary attempt to apply interval indexed fuzzy matrix theory to linear models of production.Such models are completely described by a matrix,whose coordinates have some economic interpretation.For example,the coordinates of this matrix may represent the amounts of goods consumed or produced by activities in the time-moment.But in practics,it is not easy to find out the exact values of these coordinates because the data from which they are determined often fail to be both exact and complete.In this paper,we make attempt to take this fact into account,assuming that for each $i=1,\ldots,m$, $j=1,\ldots,n$ and for any time-moment we know only a real interval.We will say,that the amounts of goods consumed or produced by activities is approximated by real intervals.In practice we do not consider all time-moment but only some time-moments.If we additionally assume that an information about choice of these time-moments is given then we shall be able to say on an fuzzy time-moment.If we also assume that an information about quality of approximation is given then we shall be able to say on the interval indexed fuzzy elements which are coordinates of our matrix. The membership function of this interval indexed fuzzy element depends on the membership function of fuzzy time-moment and on the informations about quality of approximation.We are thus led to an internal indexed fuzzy matrix which describes an interval indexed linear model of production.

In the section 3 we present the basic definitions of indexed and interval indexed fuzzy theory which we are use in the next sections.In the sections 4 and 5 we consider the simple and Leontief interval indexed fuzzy models respectively.We obtain the fuzzy analogon of Samuelson - Koopmans - Arrow substitutability theorem.

2. Notations.

iA - an interval indexed fuzzy matrix,

A - an indexed fuzzy matrix,

$A \in iA$ - A is an indexed fuzzy matrix such that ij-th coordinate of A

know only a real interval. We will say, that the amounts of goods consumed or produced by activities is approximated by real intervals. In practice we do not consider all time-moments but only some time - moments. If we additionally assume that an information about choice of these time-moments is given then we shall be able to say on an fuzzy time-moment. If we also assume that an information about quality of approximation is given then we shall be able to say on the interval indexed fuzzy elements which are coordinates of our matrix. The membership function of this interval indexed fuzzy element depends from the membership function of fuzzy time-moment and from the informations about quality of approximation. We are thus led to an interval indexed fuzzy matrix, which describes an interval indexed linear model of production.

In the section 3 we present the basic definitions of indexed and interval indexed fuzzy theory which we are use in the next sections. In the sections 4 and 5 we are conserned with the simple and Leontief interval indexed fuzzy models respectively. We obtain the fuzzy analogon of Samuelson - Koopmans - Arrow substitutability theorem. Finally, in section 6 using the connection between indexed fuzzy matrix and indexed fuzzy cone we are able to investiget von Neumann's indexed fuzzy model of an expanding economy in an explicite way.

2. Notations.

iA - an interval indexed fuzzy matrix,

A - an indexed fuzzy matrix,

if_v^{ij} - ij-th coordinate of the interval indexed fuzzy matrix,

f_v^{ij} - ij-th coordinate of the indexed fuzzy matrix,

$A \in iA$ - A is an indexed fuzzy matrix such that ij-th coordinate of A is an element of if_v^{ij} , $i=1,\dots,m$, $j=1,\dots,n$,

$iy=[if_v^1,\dots,if_v^n]$ - the interval indexed fuzzy vector whose coordinates are the interval indexed fuzzy elements,

$y=[f_v^1,\dots,f_v^n]$ - the indexed fuzzy vector whose coordinates are the indexed fuzzy elements,

$y\in iy$ - y is an indexed fuzzy vector such that i-th coordinate of y is an element of if_v^i , $i=1,\dots,n$.

3. Basic definitions of the interval indexed fuzzy sets theory.

Let T be a subset of real numbers R and let $G_T^{\langle 0,1\rangle}$ denotes the family of all functions $g : T\to\langle 0,1\rangle$. Let Y denotes arbitrary, but for further considerations fixed set. Next P(Y) denotes the family of all non-void subsets of Y. Let F be a mapping from T to P(Y). So, $\forall\, t\in T$ $F(t)\subset Y$. Instead of F(t) we will write F_t.

Definition 3.1. A generalized Cartesian product of the sets F_t ($t\in T$), F(T) say, is the set of all functions $f : T\to Y$ such that $f(t)\in F_t$, $\forall\, t\in T$.

Definition 3.2. An index fuzzy subset, v say, is a function from $G_T^{\langle 0,1\rangle}$.

The set T and the fuzzy subset v we will call the set of time-moments and the fuzzy time-moment respectively.

Definition 3.3. An indexed fuzzy subset of F(T), A_v say, is a mapping $\mu_{A_v} : F(T)\to G_T^{\langle 0,1\rangle}$ such that

(i) if $v(t)=0$ then $\mu_{A_v}(f)(t)=0$, $\forall\, f\in F(T)$, $\forall\, t\in T$,

(ii) if there exists an element $t\in T$ such that $f'(t)=f''(t)$ then $\mu_{A_v}(f')(t)=\mu_{A_v}(f'')(t)$, $f',f''\in F(T)$.

The special case of indexed fuzzy subset is an indexed fuzzy element. Namely, by an indexed fuzzy element, f_v say, it is understood an indexed fuzzy subset of $F(T)$ such that $\forall f' \in F(T)$

$$\mu_{f_v}(f') = \begin{cases} r & \text{if } f'=f, \\ 0 & \text{otherwise}, \end{cases}$$

where $r \in G_T^{\langle 0,1\rangle}$ and if $v(t)=0$ then $r(t)=0$.

Let us consider two functions $\underline{f}$ and $\overline{f}$ from $F(T)$. Let if denotes interval $if = \{ f \in F(T) : \underline{f} \leq f \leq \overline{f} \}$. We will write $if = \langle \underline{f}, \overline{f} \rangle$. If if is an interval then for any $t \in T$ $if(t) = \langle \underline{f}(t), \overline{f}(t) \rangle$ also will be an interval.

We say that a function $f \in F(T)$ belongs to an interval $if = \langle \underline{f}, \overline{f} \rangle$, $f \in if$ say, iff for any $t \in T$ $f(t) \in \langle \underline{f}(t), \overline{f}(t) \rangle$.

By the symbol $IF(T)$ we will denote the family of all intervals in $F(T)$.

Definition 3.4. An interval indexed fuzzy subset of $IF(T)$, IA_v say, is a mapping $\mu_{IA_v} : IF(T) \to G_T^{\langle 0,1\rangle}$ such that

(i) if $v(t)=0$ then $\mu_{IA_v}(if)(t)=0$, $\forall if \in IF(T)$, $\forall t \in T$,

(ii) if there exists an element $t \in T$ such that $if'(t)=if''(t)$ then $\mu_{IA_v}(if')(t) = \mu_{IA_v}(if'')(t)$, $if', if'' \in IF(T)$.

An interval indexed fuzzy element we define in the same way as an indexed fuzzy element.

We will write that $f_v \in if_v$ iff $\mu_{f_v}(f) = \mu_{if_v}(if)$ and f if, where if_v denotes an interval indexed fuzzy element.

Definition 3.5. An m-indexed fuzzy vector, a say, is an ordered set of m-indexed fuzzy elements $f^1_{v^1}, \ldots, f^m_{v^m}$.

We shall use the notation $a=(f^{i}_{v^{i}})$, meaning a is the m-indexed fuzzy vector whose i-th coordinate is $f^{i}_{v^{i}}$.

If $\mu_{f^{1}_{v^{1}}}(f^{1})=\ldots=\mu_{f^{m}_{v^{m}}}(f^{m})$ then such indexed fuzzy vector we will call the indexed fuzzy point, in symbol $\{a\}$.

Definition 3.6. An m n - indexed fuzzy matrix is a rectangular array of the indexed fuzzy elements $f^{ij}_{v^{ij}}$ (i=1(1)m, j=1(1)n).

Thus

$$A=\begin{bmatrix} f^{11}_{v^{11}} & \ldots & f^{1n}_{v^{1n}} \\ \ldots & \ldots & \ldots \\ f^{m1}_{v^{m1}} & \ldots & f^{mn}_{v^{mn}} \end{bmatrix}$$

Instead of writing out the above tableau, we will simply write $A=(f^{ij}_{v^{ij}})$, to be read "A is the indexed fuzzy matrix whose ij-th coordinate is $f^{ij}_{v^{ij}}$.

If in the above definitions instead of indexed fuzzy elements we have interval indexed fuzzy elements then we will obtain the definitions of the interval indexed fuzzy vector, interval indexed fuzzy point and interval indexed fuzzy matrix respectively.

4. The simple interval indexed fuzzy model of production.

Consider a production system, say a factory , in which n goods $T_1,\ldots,T_n$ are involved either as inputs to the productive process or as final goods. Let us assume that they have a various preferences grades.

Definition 4.1. An interval indexed fuzzy activity P involving n goods corresponds to an interval indexed fuzzy vector $ia=(if^{i}_{v})$.

The good T_i is called an input to the activity in a time moment t if $if^i(t)$ is a negative interval, and an output of the activity if $if^i(t)$ is a positive interval and $\mu_{if_v^i}(if^i) > 0$.

An interval indexed fuzzy production model IFPM involving n goods consists of a set of such activities $P_1, \dots, P_m$. Such a model is completely described by an interval indexed fuzzy matrix $iA=(if_v^{ij})$, where $if_v^{ij}(t)$ is the approximate amount of T_j produced (or consumed) in moment t when P_i is operated at unit level. The interval indexed fuzzy matrix iA is called the production matrix of the model IFPM. A production schedule for IFPM is defined to be an indexed fuzzy point with the nonnegative support. The coordinates of this indexed fuzzy point are called intensities for the activities P_i. If $\{a\}$ denotes a production schedule then $\mu_{\{a\}}(a)(t)$ denotes its quality in time moment t.

A simple interval indexed fuzzy model of production, SIFPM say, is a special case of the IFPM. The special assumptions are these:

Assumption I. Each activity P_i produces only one good T_j. In terms of interval indexed fuzzy matrix iA this assumption means that there is only one interval indexed fuzzy element with positive support in each row, all the rest being zero or negative.

Assumption II. Each good T_j is produced by one and only one activity P_i.

This means, in particular, that there are the same number of activities as goods, and it is natural to label goods and activities correspondingly. We shall agree henceforth that P_i is the activity which produces T_i. The production matrix iA for SIFPM is a square matrix.

Because of Assumptions I and II it is convenient to modify slightly the definition of the interval indexed fuzzy matrix iA as follows:

Let us agree that $if^{ij}(t)$ shall stand for the approximate amount of T_j which is necessary to consume in order to produce one unit of T_i in the time moment t.
Since consumption is now being taken as positive intervals rather than negative it is appriopriate to refer to iA as the input coeficient interval indexed fuzzy matrix of the model. The interval indexed fuzzy matrix iA is called the consumption matrix of SIFPM.

Suppose that model with iA is asked to produce ih^1 units of T_1, which has a quality $\mu_{ih^1_V}(ih^1)$, ih^2 units of T_2, which has a quality $\mu_{ih^2_V}(ih^2)$ etc. Let $ib=(ih^i_V)$ and let the model is operated at level $\{a\}$. Then the amounts consumed by the whole model will be

$$\{a\}\cdot iA = \{\{a\}\cdot A : A \in iA\}$$

and the net production that is gross outpup minus input requirement is given by the set

$$\{\{a\} - \{a\}\cdot A : A \in iA\}.$$

The feasibility question is simply: Given ib with a nonnegative support does the equation

$$\{a\} - \{a\}\cdot A = b$$

have a solution with a nonnegative support for some $A \in iA$ and for all $b \in ib$? .

Definition 4.2. A simple interval indexed fuzzy model with input coeficient interval indexed fuzzy matrix iA will be called productive if there exists $\bar{A} \in iA$ and an indexed fuzzy point $\{\bar{a}\}$ with the nonnegative support such that

$$\{\bar{a}\} > \{\bar{a}\}\cdot\bar{A} .$$

In this case we shall say that iA itself is productive with regard to the indexed fuzzy matrix $\bar{A}$.

Theorem 4.1. If the interval indexed fuzzy matrix iA is productive with regard to an indexed fuzzy matrix $\bar{A}$ then for any indexed fuzzy vector b with the nonnegative support the indexed fuzzy equation

$$\{a\} - \{a\}\cdot\bar{A} = b$$

has a unique solution with nonnegative support, on the condition that

$$\forall j \quad \iota_{h^j}(h^j) \,/\, \underset{i}{TN} \; f_v^{ij}(\bar{f}^{ij})\cdot\beta \in 0_T^{<0,1>},$$

where TN denotes the indexed t-norm (compare Matłoka (1984)).

The theorem will be a consequence of the following lemma.

Lemma 4.1. If iA is productive with regard to $\bar{A}$ and $\{a\} \geq \{a\}\cdot\bar{A}$ then $\forall i$ supp $f_v^i \geq 0$, where $\{a\} = (f_v^i)$.

Proof. By definition there exists an indexed fuzzy point $\{\bar{a}\} = (\bar{f}_v^i)$ with the nonnegative support such that $\bar{a} \quad \bar{a}\;\bar{A}$. This means that $\bar{f}^i > 0$, $\forall i$. Suppose now that $\{a\} = (f_v^i)$ and $\{a\} \geq \{a\}\cdot\bar{A}$ and $f^i \ngeq 0$. Then at least one i and $\bar{t}$ $f^i(\bar{t}) < 0$. Let $\alpha = \max(-f^i(\bar{t})/\bar{f}^i(\bar{t}))$, say $\alpha = -f^1(\bar{t})/\bar{f}^1(\bar{t})$. Then α is positive and $\{a'\} = \{a\} + \alpha\{\bar{a}\}$ has the nonnegative support with $f'^1(\bar{t}) = 0$. But also

$$\{a'\} = \{a\} + \alpha\{\bar{a}\} > (\{a\} + \alpha\{\bar{a}\})\,\bar{A} \geq 0_c\,,$$

which would imply

$$f'^1(\bar{t}) > \sum_{i=1}^{m} f'^i(\bar{t})\cdot f^{i1}(\bar{t}) \geq 0,$$ a contradiction (0_c - zero indexed fuzzy vector).

Let us note that $\{a\} - \{a\}\cdot\bar{A}$ we can write in the following form

$$\{a\}\cdot(I - \bar{A}),$$

where I is an indexed fuzzy matrix with the elements 1_v^{ij} such that

$$1^{ij} = \begin{cases} 1_f & \text{if } i=j, \\ 0_f & \text{if } i \neq j, \end{cases} \quad , \quad 1^{ij} : T \to F(T),$$

and $\forall t \in \operatorname{supp} v \quad \mu_{1_v^{ij}}(1^{ij})(t)=1.$

Corollary. If iA is productive with regard to $\overline{A}$ then the matrix $(I - \overline{A})(t)$ has rank n for all $t \in T$, where $(I - \overline{A})(t)$ is the matrix with the elements $(1^{ij}(t) - \overline{f}^{ij}(t))$.

Proof of the Theorem. Because $(I - \overline{A})(t)$ has rank n and $\forall j$ $\mu_{h_v^j}(h^j)/\underset{j}{TN}\,\mu_{\overline{f}_v^{ij}}(\overline{f}^{ij})=\beta \in \quad G_T^{<0,1>}$ then there exists a unique indexed fuzzy point $\{a\}$ such that $\{a\}(I - \overline{A})=b$ and since $h^j \geqslant 0_f$ the lemma implies $f^i \geqslant 0_f$, $\forall i$.

So far our discusion has been concerned entirely with the quantity side. We turn now to the price side by introducing prices. As usual we let $\{p\}=(p_v^j)$ be the indexed fuzzy point (fuzzy price vector). Then the profit is given by the set

$\{\{p\} - A\cdot\{p\} : A \in iA\}$.

Theorem 4.2. If iA is productive with regard to $\overline{A}$ then for any indexed fuzzy vector $q=(k_v^i)$ with the nonnegative support such that $\forall i$

$$\mu_{k_v^i}(k^i)/\underset{j}{TN}\,\mu_{f_v^{ij}}(f^{ij})= \beta \in \quad G_T^{<0,1>}$$

there exists a unique indexed fuzzy element $\{p\}$ with the nonnegative support such that

$q=(I - \overline{A})\cdot\{p\}$.

The proof is a consequence of the Theorem 4.1.

5. The interval indexed fuzzy Leontief model.

A simple interval indexed fuzzy model - as can observe - has the inconveniences as a closed one. Therefore, we in our SIFPM modifications analogous to these ones which are used when we are passing from the closed simple model to the open Leontief model. Then the interval $if^{ij}(t)$ becomes the approximate amount of T_j required per t in order to obtain in that time an output of one unit of T_i .

Definition 5.1. The simple interval indexed fuzzy Leontief model consists of a simple interval indexed fuzzy model in which is a single primary good T_0 called labor.

We shall assume that labor is needed as input to all activities, that is, the consumption intervals $if^{i0}(t)$ are all positive ($t \in \mathrm{supp}\,v$) For the interval indexed fuzzy Leontief model with labor as primary in input is natural to assume that when the cost of labor is taken into account the profit of each activity shall be zero.

Theorem 5.1. Let iA be a productive matrix with regard to $\bar{A}$. Then there exists an indexed fuzzy point $\{p\}$ with the positive support, unique up to multiplication by the ~~activity~~ positive functions, such that at prices $\{p\}$ the profit to each activity is zero (on the condition that $\forall i \quad \mu_{\bar{I}^{i0}_v}(\bar{I}^{i0}) = \alpha \cdot TN_{j>0}\, \mu_{\bar{I}^{ij}_v}(\bar{I}^{ij})$). The multiplication ~~refer text~~
refer to 1-multiplication and 2-multiplication by a function $\alpha : t \rightarrow \langle 0, 1 / TN_{i,j}\, \mu_{\bar{I}^{ij}_v}(\bar{I}^{ij}) \rangle$ (compare Matłoka (1984)).

Proof. Let iA is productive with regard to $\bar{A}$ and let for $\{p\} = (g^i_v)$, $g^0 = 1_f$. The condition that profits be zero is then

$$(A - A')\{p\} = a^0, \qquad (*)$$

where $a^0 = (i^{i0}_v)$ and A' is the indexed fuzzy matrix without the column a^0. By theorem 4.1 (*) has a unique solution with the nonnegative support and since support a^0 is positive, $\{p\}$ also has the positive support.

Definition 5.2. A general interval indexed fuzzy Leontief model is the interval indexed fuzzy Leontief model to satisfy all the conditions imposed on the simple model except that the good T_j may be producible by more than one activity.

We denote by S_j the set of all indices i such that P_i produces T_j . Let $\{a\} = (f^i_v)$ be an intensity vector. Then an indexed fuzzy vector of net output $b=(h^j_v)$ is given by

$$h^j_v = \sum_{i \in S_j} f^i_v - \sum_{i=1}^{m} f^i_v \cdot \bar{f}^{ij}_v ,$$

where

$$\sum_{i=1}^{m} f^i_v \, \bar{f}^{ij}_v \leq 1_c .$$

The set H of all such indexed fuzzy vector will be termed the output space of the model.

Theorem 5.2. If a general interval indexed fuzzy Leontief model is productive with regard to $\bar{\Lambda}$ then there exists a set of n activities $P_{i_1}, \ldots, P_{i_n}$, where $i_j \in S_j$, such that the interval indexed fuzzy Leontief model formed from these activities has same output space for $\bar{\Lambda}$ as the original model.

Let us reconsider the canonical fuzzy problem (compare Matłoka (1984)) of finding an indexed fuzzy point $\{a'\}$ with the nonnegative support such that

$$\{a'\} \cdot c \leq \{a\} \cdot c, \qquad \forall a \in F(T) \times \ldots \times F(T) \qquad (1)$$

$$\{a\} \cdot A = b \qquad (2)$$

Definition 5.3. We call a set of independent rows a_i of an indexed fuzzy matrix A an optimal (feasible) basis if there is an optimal (feasible) indexed fuzzy point depending on these rows.

Lemma 5.1. Let a set B of rows a_i of indexed fuzzy matrix A be an optimal basis for the problem (1), (2) above and consider the new problem : find $\{a''\}$ with the nonnegative support such that

$$\{a''\} \cdot c \leq \{a\} \cdot c \qquad \forall a \in F(T) \times \ldots \times F(T) \qquad (1')$$

$$\{a\} \cdot A = b' \tag{2'}$$

where $G=(g^i_v)$. Then if B is a feasible basis for problem (1'),(2') it is, in fact, an optimal basis for this problem also.

Proof. Let $\{a'\}$ be an optimal indexed fuzzy point for problem (1),(2) depending on the basis B and $\{d'\}$ be a solution of the dual. Then we know (compare Mařoka (1984)) that

$$\text{if } a_i \cdot \{d'\} > g^i_v \quad \text{then} \quad f^{i\prime} = 0_f \tag{*}$$

Now assume $\{\bar{a}'\}$ is a feasible indexed fuzzy point of problem (1'),(2') depending on the set B. Then we have also

$$\text{if } a_i \cdot \{d'\} > g^i_v \quad \text{then} \quad \bar{f}^i = 0_f \tag{*'}$$

But this is precisely the condition that $\{\bar{a}'\}$ and $\{d'\}$ be solutions of the primal and dual problems of (1') and (2') and, in particular, $\{\bar{a}'\}$ is an optimal indexed fuzzy vector, as asserted.

Proof of the Theorem 5.2. Let $b=(h^j_v)$ be an indexed fuzzy vector with the positive support in the output space W. We consider the canonical fuzzy problem of producing b while minimizing the amount of labor used, that is,

$$\{a'\} \cdot a^0 \leq \{a\} \cdot a^0 \qquad \forall a \in F(T) \times \dots \times F(T) \ .$$

Now, let $\{a'\}$ be a basic optimal indexed fuzzy point for this problem. Then $\{a'\}$ depends on at most n rows $i_1, \dots, i_n$ of the indexed fuzzy matrix A. Because $h^j > 0_f$, so $f^{i\prime} > 0_f$ for one index i in each of the sets S_j. Letting A' be the indexed fuzzy matrix with rows a_{i_j} it remains to show that A' has the output space W. Let $b' \in W$. Because $h^j > 0_f$ so A' is the productive matrix. Hence, by Theorem 4.1 , there exists an indexed fuzzy point $\{\bar{a}'\}$ such that

$$\{\bar{a}'\} \cdot (I - A') = b' \ ,$$

but this simply says that the basis given by the rows a_{i_j} is feasible for the new program, where vector to be produced is b' rather b. According to the Lemma 5.1 , $\{\bar{a}'\}$ is also optimal, that is

$\{\overline{a}'\} a^0 \leq \{a\} \cdot a^0$

among all possible $\{a\}$ for the original model. Since $b' \in W$ there is some $\{a\}$ such that $\{a\} \cdot A' = b'$ and $\{a\} \cdot a^0 \leq 1_c$. It now follows that

$\{\overline{a}'\} \cdot A' = b'$

because

$\{\overline{a}'\} \cdot a^0 \leq 1_c$.

The proof is now complete.

References

[1] Albrycht,J and Matłoka,M : On fuzzy multi-valued functions, Part 1: Introduction and general properties, Fuzzy Sets and Systems 12 (1984), 61-69.

[2] Matłoka,M : On indexed fuzzy subsets, Busefal 18 (1984), 42-52.

[3] Matłoka,M : On fuzzy vectors and fuzzy matrices, Busefal 19 (1984), 92-105.

[4] Matłoka,M : On fuzzy programming, Busefal 20 (1984).

[5] Matłoka,M : An interval deterministic model of the economical dynamics, (in Polish), Zeszyty Naukowe Akademii Ekonomicznej w Poznaniu 94 (1981), 146-154.

[6] Rohn,J : Input-output planning with inexact data, Freiburger Interval - Berichte, 1978/79.

[7] Zadeh,L.A : Fuzzy sets, Information and Control 1965, 8, 338-353.