

Book Review

Probabilistic sets in decision-making and control, by E. Czogala
 Interdisciplinary Systems Research vol 81, Verlag TÜV, Rheinland,
 Köln, 1984, 113 p., (reviewed by D. Dubois).

A probabilistic set is a fuzzy set with a random membership function. This concept was introduced by Hirota in 1977, as an attempt to cope with the problem of membership function estimation. This model is natural when the membership function reflects the opinions of a group of people, the membership grade of each element in the reference set then taking the form of a histogram over the valuation set (i.e. the unit interval).

This short monograph proposes an exhaustive presentation of probabilistic sets and their existing applications. It is divided into 4 chapters; the two first are devoted to the theory, the two last to applications. The first chapter gives the mathematical definition of probabilistic sets and their operations. The set of probabilistic subsets of a reference set is a complete distributive lattice with pseudo-complement, if the max-min theory of fuzzy sets is used as a basis. Chapter 2 takes advantage of the distribution function of the membership grades to derive expressions of probabilistic-set operations for the purpose of actual computation. Chapter 3 is a straightforward extension of Bellman and Zadeh's [2] paradigm of decision-making, when goals and constraints are viewed as probabilistic sets. Using the max-min setting, the results which are obtained (i.e. the expected optimal decision) differ from those of the Bellman and Zadeh's approach because the expected membership function $E(\mu_{A \cap B})$ is not $\min(E(\mu_A), E(\mu_B))$. The last chapter is an extension of Mamdani's fuzzy rule-based controllers, where involved fuzzy sets are changed into probabilistic sets. Some considerations about the identification problem (i.e. find the fuzzy system from input-output pairs) are briefly provided.

The idea of probabilistic sets is an interesting one, intuitively. However, the reader gets some how frustrated reading this monograph for several reasons. First there is no attempt to compare Hirota's proposal with the ones of other people interested in similar questions, for instance what Zadeh calls ultra-fuzzy sets (i.e. fuzzy sets with fuzzy membership grades. [3]), Kaufmann [4] random fuzzy numbers, Kwakernaak [5]'s fuzzy random variables etc ..., although most of

these works appear in the bibliography. Another point is that the discussion about the difference between fuzziness and randomness appears not to be developed enough, which may discourage readers outside the fuzzy community, facing a concept which mixes both notions. From the point of view of the potential users of probabilistic sets, they may be deterred from choosing this approach, due to its computational difficulties. Operations on probabilistic sets are based on the theory of functions of random variables. Calculating with random variables is not always straight-forward. Lastly we may question the nature of the proposed applications : basically why would you use probabilistic sets instead of fuzzy sets in decision-making and control ? This question receives but little attention in the monograph. This may be viewed as problematic when on the other hand the usefulness of fuzzy sets in decision-making and control is, to-date, not self-evident for many people outside the fuzzy community (e.g. French [6]). To this respect, the examples of decision-making and control problems in the book are very academic, thus rather unconvincing. May be it would have been better to present probabilistic sets as such, i.e. an intuitively appealing notion with a rigorous mathematical treatment ; may be is it too early to provide a definite assessment of their usefulness in practical problems. In spite of these reserves this book is worth reading by researchers in the fuzzy area, due to the natural appeal of the concept.

- 1 - Hirota K. Concepts of probabilistic sets. Fuzzy Sets & Systems. 5, 31-46, 1981.
- 2 - Bellmann R.E., Zadeh, L.A. Decision-making in a fuzzy environment, Management Science 17, 149-156, 1970.
- 3 - Mizumoto, M., Tanaka, K. Some properties of fuzzy sets of type 2, Inf & Control 31, 312-340, 1976.
- 4 - Kaufman, A. Hybrid Convolution. Fuzzy Math. (China) vol. 1, n° 2.
- 5 - Kwakernaak, H. Fuzzy Random variables. Inf. Sciences 15, 1-29, 1978, and 17, 253-278, 1980.
- 6 - French, S. Fuzzy decision-making : some criticisms in Fuzzy Sets and Decision Analysis (H-J. Zimmermann et al. Eds) TIMS Studies in The Management Sciences, vol. 20, North-Holland, p. 29 et seq ..., 1984.