METHOD OF MATTER ELEMENT ANALYSIS

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ABSTRACT

In the real life many problems appear to be unsolvable, but can be made solvable through some methods of transformation. For example, it is unpossible for one to utilize a steelyard to weigh an elephant. However, the story of "Tsao Chung weighs elephant" revealed a possible way of solving this weighing problem. The basic method called the method of matter element analysis and the main steps for Solving similar incompatible problems are introduced in this article.

KEYWORDS

Incompatible problem; extension set; matter element; mathod of matter element analysis.

§ 1. Incompatible problem

In our lives, We always meet with many rather difficult problems concerning our work and scientific research. No matter whether ancient or modern, chinese or foreigners, in-numerable difficult problems baffled many people. But some of them had been solved with the help of some good suggestions. For example, It is impossible for anyone to weigh an elephant with only a steelyard. However, Chao Chong, a seven-year old boy, did think out a good idea to solve this problem: The elephant was first driven on a boat, then he marked a waterline on both sides of the boat, After that, the elephant was driven away from the boat, and then stones were loaded one by one until the waterline markd on the boat were of the same as before. Then the total weight of the stones was the weight of the elephant.

In ancient Europe, there was a similar story, The story was about how Archimedes estimated the volume of a golden imperial crown. He put the crown into a container which was previously filled with water. Then the volume of the overflowing water Should be equivalent to the volume of the imperial crown.

In real life, We meet with similar problems. For example, in order to move a machine which is taller than the door of the workshop, we usually use the methos of disassembling the machine, separating the machine into several parts, then assembling the machine in the workshop. In chemical reactions, some catalyst can be added to some ehemical compounds in order to make the reaction possible. In engineering design, one can use equicohesive project to solve contradictions arising from different strengths on the work-piece.

We have found that these incompatible problems have a common characteristic that "goal to effect" can not be realized by the conditions given, but such kinds of problem can be turned into compatible ones by means of some clever methods.

Some can find a method for solving difficult problems but not all can do so. Is there any theory or law for solving difficult problems? Can we solve them by computer? This new task has been dealt with briefly in Art [1] . The basic method called the method of matter element analysis and the main steps for solving incompatible problems are introduced in this article.

Noblist professor H. A. Simon said:"I found your fomulation of the problem interesting and your examples helpful."

"Matter Element Analysis" Consists of "Theory of matter element", "Method of matter element analysis". "Theory of Extension Set". "Experimental method of matter element analysis".

Engineering, scientific research, militar gaming and ecomonic administration provide a lot of row materials and are open to its practice. Nowadays, many incompitible problems can be solved by the concept of matter element. Incompatible problems will be solved by computer.

§ 2. Fundamental Tools of Matter Element Analysis

Some basic concepts of matter element analysis have already been established both in the Art [1] and [2].

1. Matter elements and their transformations. We used an orderly three-dimensional group

$$r=(M, C, X)$$

as the fundamental element to discribe the matter, and call it a "Matter Element" M shows the matter, C shows the characteristics (i, e length, volume, mass, colour etc) and X shows the measure of M about C. If a matter is descrided by n Characteristics, then an n-dimensional matter element can be made up for example,

$$R=(M,\ C,\ X)=\begin{pmatrix} M, & C_1, & X_1 \\ & C_2 & X_2 \\ \cdots\cdots & & \\ & C_n & X_n \end{pmatrix}$$

$$=\begin{pmatrix} \text{machine}, & \text{weight} & 50\ \text{ton} \\ & \text{length} & 12\ \text{m} \\ \cdots\cdots\cdots & & \\ & \text{colour} & \text{blue} \end{pmatrix}$$

is an n-dimensional matter element. $(M,\ C_i,\ X_i)$ $(i=1,\ 2,\ \cdots\ n)$ is called "the branch matter element of R".

The method by which a matter element is turned into another matter element or several matter elements $r_1,\ r_2,\ \cdots,\ r_n$ is called the matter element transformation. A matter element transformation may be transformation of matter, characteristic, or measure, written as:

$$(T_M,\ T_C,\ T_X)$$

Then

$$T_{r_0}=(T_M,\ T_C,\ T_X)\ (M_0,\ C_0,\ X_0)$$
$$=(T_M\ M_0,\ T_C\ C_0,\ T_X\ X_0)$$

All the matter, characteristic and measure of a matter element have the following four fundamental transformations:

1. Combine (or add)
2. resolve (or cut)
3. similar (or enlarge, or reduce)
4. replace (or move)

At the same time, the transformation of the matter element also has the following four basic operations:

1. Product $T = T_2 \cdot T_1$
2. Converse $T^{-1}T = E$
3. And $T = T_1 \otimes T_2$
4. Or $T = T_1\ \&\ T_2$

Any transformations may be considered as the fundamental transformation of the matter element obtained by transformation operations.

2. Extension set and Dependent function ______ the fundamental matter analysis.

In classical mathematics, a true thing is true and a false thing is false. People use two numbers $\{0,1\}$ to indicate whether an element belongs to a set or not.
In the Fuzzy set $[0,1]$ is taken as the values of the degree of membership.
In order to discuss the outer element of a set which can be transformed into the inner element of the sets, the concept of the extension set is established in Art [1]. The real number set $(-\infty, +\infty)$ stands for the degree of the thing that belongs to the extension set. We use a real number to measure the relation of an element and the set. The mapping:

$$u \rightarrow (-\infty, +\infty)$$

$$K_{\tilde{X}}: \quad u \rightarrow K_{\tilde{X}}(u)$$

is called a dependent function of $\tilde{X}$.
The extension set and the dependent function are the basic concepts for solving incompatible problems.
All kinds of real background and new mathematical tools can be set up through the concept of extension set, such as extension geometry and extension matrix etc.
Because there are many problems which refer to a constant described by a real number, we partioularly discussed the extension set and the dependent function of the real number field in Art [1] and established

$$K(y) = \frac{(y, x_0)}{p(y, x_0 - x) - p(y, x_0)} .$$

This is a very important dependent function.
3. Problems and their operations.
What does a question P mean? That is to say, a goal must be realized through some conditions.
R_0 stands for the goal matter element, r_0 for the condition matter element and problem P written as

$$P = R_0 * r_0 .$$

The problem consisting of the goal the condition which can be realized is called the compatible problem. The problem which consists of the goals matter element and the condition which makes the goal matter not be realized is called the incompatible problem.
In the real world, a problem is rather complex. A problem consisting of an m-dimensional goal matter element and an n-dimensional condition matter element is called a fundamental problem.

A fundamental problem which is made up of a 1-dimensional goal matter element and a 1-dimensional condition matter element, is called a simple problem. Any complex problem can be regarded as a fundamental problem obtained by some ways of logical operations.

Two sorts of solving the basic problem have been found in Art [1].

1. The single direction transformation.

If an incompatible problem is solved by changing the goal matter element and the condition matter element, after solving problem, the original matter element need not be restored, this transformation is called a single direction transformation.

2. The succesive negative transformation.

After solving an incompatible problem, the original matter element must be restored, we call this the succesive negative transformation.

§ 3. Basic Steps Of The Matter Element Analysis.

To solve an incompatible realistic problem with matter element is a new thing. We first resoive a complex problem into a few simple problems, and solve them with methos of matter element analysis, then combine the solutions of these problems as the solution of the original problem. When we solve a simple problem, we must consider the change of condition and the goal at the same time. That is to say, when the goal is not realized by the known condition, the incompatible problem can be solved by the changing condition, or by realizing the goal which can be exchanged into the original goal.

We use qualitative analysis to solve each simple incompatible problem in order to get the sorts of transformation, and use quantitative computational method to get a certain quantity, and then the optimum solution is succesfully chosen through compaison of the optimal degree of solutions. The above procedure has been programmed in Art [4]. This intellingent method may be advanced.

The basic steps of the matter element analysis are as follows.

1. To turn the problem into matter elements, and write down the logical formulae.

At first we list the logical formulae of the goal matter element and the condition matter element.

$$R = L\ (R_1, R_2, \cdots, R_n)$$

and

$$r = L(r_1, r_2, \cdots, r_n)$$

then, the complex problem may be written as

$$P = R * r$$

$$= L(R_1, R_2, \cdots, R_m) * L(r_1, r_2, \cdots, r_n)$$

Because of the logical property of the problem operations, it is converted into the model formula

$$P = L(P_1, P_2, \cdots, P_k)$$

Where $P_1, P_2, \cdots, P_k$ are all basic problems.
If the goal matter element is the sum of a few matter elements, the original problem can be divided into the sum of a few matter elements, written as:

$$(R_1+R_2)*r = R_1*r+R_2*(r-r_1)$$

Where r_1 is the needful condition by which R_1 can be realized.
2. To solve the fundamental problem.
1) To complete the compatible degree $K_r(R)$ by means of solving the dependent inegualiity.
If $K_r(R)>0$, P is a compatible problem. If $K_r(R)\leq 0$, P is a incompatible problem.
2) To establish the dependent inequality.
If $K_r(R)<0$, following dependent inequality

$$K_{T_r \cdot r}(R)>0$$

$$K_r(T_R \cdot R)>0$$

$$K_{T_r \cdot r}(T_R \cdot R)>0$$

are established for the incompatible problem. That is to say, the problem may be solved by means of the turning condition or the goal or both at the same time.
3) To solve the dependent inequality.
To solve the dependent inequality means to find the sorts and the measure of transformation which convert the incompatible problem into the compatible one. This sort of transformation is obtained by qualitative analysis, and the measure of the transformation is obtained by a quantitative operation.
A) Qualitative Analysis.
First, the matter, the characteristic and the measure of the matter element are analyzed one by one. According to the restriction of the problem, the use of four fundamental transformation is decided one by one. The one which can be used is malked by "O", Otherwise the it is marked

by "X":

Matter M: a b c d — x o x x

Characteristics C: a b c d — o o x x

Measure X: a b c d — x o x x

Where

a —— combine
b —— resolve
c —— similar
d —— replace

B) Qualitative computation.

The measure of a transformation is computed by the dependent function:

$$K_{T_r}(R)>0$$

$$K_r(TR)>0$$

$$K_{T_r r}(T_R R)>0$$

3. Combination —— to combine the transformation of these basic problems. If $\{T_i\}$ is a set of solution tramsformations of the problems

$$P_i = R_i * r_i$$

then a logical formula of the transformations

$$L(\{T_1\}, \{T_2\}, \cdots, \{T_n\})$$

can be written according to logical formula of the problem

$$P = L(P_1, P_2, \cdots, P_n)$$

This is a set of the solution transformation. In this transformation set some matter element does not allow us to use some transformations simultaneous. So we must discuss the set $L(\{T_1\}, \{T_2\}, \cdots, \{T_k\})$.

If there are $T_i \in \{T_i\}$ $(i=1, 2, \cdots, k)$,

That $L(T_{10}, T_{20}, \cdots, T_{k0})$ can be realized, then

$$T_0 = L(T_{10}, T_{20}, \cdots, T_{k0})$$

is a solution transformation of the problem. Clearly T_0 is not the only one. They are formed a set $\{ T\}$.

4. To decide the optimum solution

Each solution of the solution set $\{T\}$ of the problem P is computed respectively and obtained the optimum degree [1].

Compared with the optimum degree of the sclution, the optimum solution is decided.

5. To program.
The method solving a kind of incompatible problem is programmed, and incompatible problems are analyzed and operated by computer.

§ 4. Practice

We give an example "Chao Chong Weighs an elephant".
1. Express the problem with the matter elements.

$$R_0 - R_0 \text{ (elephant, weight, x)}$$
$$r_0 - r_0 \text{ (steelyard, weighing region, (0, 200))}$$

Then

$$P = R_0 * r_0$$

P is a fundamental problem.
2. To compute a compatible degree.

$$\because X_0 = (0, 200)$$
$$X = \phi$$

According to the theory of Art [1], the compatible degree is

$$\begin{aligned} K_r(R) &= -p(X, X_0) - 1 \\ &= -\left|X - \frac{200}{2}\right| - \left|\frac{200}{2}\right| - 1 \\ &= 99 - |X - 100| \\ &< -1 \end{aligned}$$

It is shown that P is a incompatible problem. The dependent inequality is established

$$K_r(TR) > 0,$$
$$K_{T_r}(R) > 0,$$
$$K_{T_r}r(T_R R) > 0.$$

3. To solve the goal inequality.
At first, the basic transformation of the goal matter element is analyzed.

elephant M	weight C	measure X
a b c d	a b c d	a b c d
x x x o	x x x x	x x x x

From the above analysis, we get the transformation T_m^d. That means the matter element R_0 (elephant, weight, X ton)

$$T_m^d R = R_1 \text{ (\{body\}, weight, X ton)}$$

The problem P is turned into

r_0 (steelyard, weighting region (0, 200))

	{body}	weight	X)
R_1 (	a b c d	a b c d	a b c d
	x o x x	x x x x	x o x x

According to above statement, we get the transformation (T_m^b, e T_X^b), So the transformation of the goal matter element is:

$$T_R = (T_m^b \; T_m^d \; , \; e, \; T_X^b)$$

It is the method that Chao Chong used.
First of all, the weight of the elephant is changed into the weight of the stone, and the stones is divided into a few piles.
Then how many times are the stones weighed? That is to say, how many piles must the stones be divided?
If the stones are divided into n piles, according to dependent inequality. We get

$$\left|\frac{X}{n} - \frac{200}{2}\right| - \left|\frac{200}{2}\right| \geqslant 0$$

then

$$h \geqslant \frac{X}{200}$$

If the weight of the elephant is about 5000 kilogram, then the stones must be weighed 25 times at least. This result is fit for logic.
4. To solve the condition inequatity (2)
At first the condition matter element is analyzed

Steelyard	Weighing region	Y(0, 200)
a b c d	a b c d	a b c d
o x x o	x x x x	x x x x

The capable transformation

(T_m^a , e, T_X^a) and (T_m^d , e, T_X^d) are got

In this way, the condition matter element is changed into

(T_m^a M, C, T_X^a Y) $\qquad$ $T_X^a\, Y > X$

and

(T_m^d M, C, T_X^d Y) $\qquad$ $T_X^d\, Y > X$

namely:

1. (T_m^a, e, T_x^a) to add a few steelyards which can weigh the elephant.
2. (T_m^d, e, T_x^d) to replace a large steelyard. (Nowadays we can use weighbridge.)

We get three capable solution transformations in all.

Example 2

Every day four large cargo ships and thirty small cargosboats need to be discharged at the wharf.
The length of a large ship is 16 metres and it has a draught of 5 metres. It takes two hours to discharge each large ship.
The length of a small boat is 11 metres and it has a draught of 3m. It takes an hour to discharge each small boat.
The width of the wharf is 20 metres. The depth of the wharf is 6 metres at high tide and 3.5 metres at ebbtide. 4 metres under normal condition.
There are two times rising tides and two times ebbing tides per-day.
The time of the tide rising and ebbing is two hours for each one.
Cargoes did not be discharged in time if the work of the wharf did not managed well. How can we manage it well and discharge the cargoes in time.

1. We try to express the problem with matter element.

$$P = R * r$$

$$R = \sum_{i=1}^{4} A_i + \sum_{i=1}^{30} \bar{A}_i = R_1 + R_2$$

where

$$A_i = \begin{pmatrix} A_i^{(1)} \\ A_i^{(2)} \\ A_i^{(3)} \end{pmatrix} = \begin{pmatrix} \text{large ship } i & \text{length} & 16\text{ m} \\ & \text{load draught} & 5\text{ m} \\ & \text{discharged time} & 2\text{ hours} \end{pmatrix}$$

$$\bar{A}_i = \begin{pmatrix} \bar{A}_i^{(1)} \\ \bar{A}_i^{(2)} \\ \bar{A}_i^{(3)} \\ \bar{A}_i^{(4)} \end{pmatrix} = \begin{pmatrix} \text{smallship } i & \text{length} & 11\text{ m} \\ & \text{load draught} & 3\text{ m} \\ & \text{discharged time} & 2\text{ hours} \end{pmatrix}$$

$$r = \begin{pmatrix} r^{(1)} \\ r^{(2)} \\ r^{(3)} \end{pmatrix} = \begin{pmatrix} \text{wharf} & \text{width} & 20\text{ m} \\ & \text{depth} & x(t) \\ & \text{discharged time} & 24\text{ hours} \end{pmatrix}$$

where

$$X(t) = \begin{cases} 0 & t \in [0, 2] \quad [12, 14] \\ 3.5 & t \in [6, 8] \quad [18, 20] \\ 4 & \text{others} \end{cases}$$

2. To compute the compatible degree and establish the dependent inequality.

$$K_r^{(1)} \quad (A_i^{(1)}) > 0$$

$$K_r^{(1)} \quad (B_i^{(1)}) > 0$$

$$K_r^{(2)} \quad (A_i^{(2)}) \begin{cases} >0 & t \in [0, 2]\ ,\ [12, 14] \\ <0 & \text{others} \end{cases}$$

$$K_r^{(2)} \quad (B_i^{(2)}) > 0$$

$$K_r^{(3)} \quad (R^{(3)}) = \left(\left|38 - \frac{24-0}{2}\right| - \frac{24+0}{2} \right) = -14 < 0$$

This is an incompatible problem —— We discuss the dependent inequality

$$K_r \ (TR) > 0$$

$$\because \quad R = \sum_{i=1}^{4} A_i + \sum_{i=1}^{30} B_i = R_1 + R_2$$

So the problem can be turned into

$$R * r = (R_1 + R_2) * r$$

$$= R_1 * r + R_2 * (r - r_1)$$

r_1 is a needful condition by which R_1 can be realized.

3. To Solve $R_1 * r$ and $R_2 * (r - r_1)$

$$R_1 = \begin{pmatrix} R_1^{(1)} \\ R_1^{(2)} \\ R_1^{(3)} \end{pmatrix} = \begin{pmatrix} \text{large ship} & \text{length} & 16 \text{ m} \\ & \text{load draught} & 5 \text{ m} \\ & \text{discharged time} & 8 \text{ hours} \end{pmatrix}$$

$$r_1 = \begin{pmatrix} r_1^{(1)} \\ r_1^{(2)} \\ r_1^{(3)} \end{pmatrix} = \begin{pmatrix} \text{wharf} & \text{width} & 20 \text{ m} \\ & \text{depth} & 6 \text{ m} \\ & \text{time in high tide} & 4 \text{ hours} \end{pmatrix}$$

clearly

$$K_{r_1}^{(1)}(R_1^{(1)}) > 0$$

$$K_{r_1}^{(2)}(R_1^{(2)}) > 0$$

$$K_{r_1}^{(3)}(R_1^{(3)}) < 0$$

The dependent inequality

$$K_T\, r^{(3)}(R_1^{(3)}) > 0$$

is first solved.
The matter element

$r^{(3)}$ = (wharf, high tide time, 4 hours)

	wharf	high tide time	4 hours
	a b c d	a b c d	a b c d
	x x x x	o x x x	o x x x

is analyzed. Namely, the addition time is 4 hours. But r (2) and r (3) are dependent. They can not be realized at high tide time together,

$$\therefore\ K_r^{(2)}(R^{(2)}) < 0\,.$$

The new incompatible problem (Large ship, draught, 5m)*(wharf, depth, 4m) appears at this time. Because the condition is unchanged. The transformation of the goal matter element is analyzed. The draught of the large ship will be changed when the ship is dischanged. So this problem must be written as:

(large ship, draught 5-1.2L)
(wharf, depth, 4 m)

Where L is dischanged time. Clearly

$$K_r^{(2)}(R^{(2)}) \begin{cases} > 0 & 5 - 1.2\,L < 4 \\ < 0 & 5 - 1.2\,L > 4 \end{cases}$$

Solving

$$5 - 1.2\,L < 4$$

getting

$$L > \frac{1}{1.2}$$

When the large ship will be finished Loading in 1 hour, the ship will sail on the wharf at ebb-tide freely.
The second, the matter element $R_1^{(3)}$ is analyzed.

(large ship	discharged time	2 hours)
a b c d	a b c d	a b c d
x x x x	x x x x	x o x x

It is shown that the discharging time can be divided into two parts. One is at the high tide, the other is at low tide.
The third, the problem $R_2 * (r - r_1)$ is solved.

$$R_2 = \begin{pmatrix} R_2^{(1)} \\ R_2^{(2)} \\ R_2^{(3)} \end{pmatrix} = \begin{pmatrix} \text{boat} & \text{length} & 11 \text{ m} \\ & \text{draught} & 3 \text{ m} \\ & \text{discharge time} & 30 \text{ hours} \end{pmatrix}$$

$$r' = r - r_1 = \begin{pmatrix} r'^{(1)} \\ r'^{(2)} \\ r'^{(3)} \end{pmatrix} = \begin{pmatrix} \text{wharf} & \text{width} & 20 \text{ m} \\ & \text{depth} & X(t) \\ & \text{discharge time} & 16 \text{ hours} \end{pmatrix}$$

clearly

$$K_{r'}^{(1)}(R_2^{(1)}) > 0$$

$$K_{r'}^{(2)}(R_2^{(2)}) > 0$$

$$K_{r'}^{(3)}(R_2^{(3)}) < 0$$

The dependent inequality

$$K_{r'}(TR_2^{(3)}) > 0$$

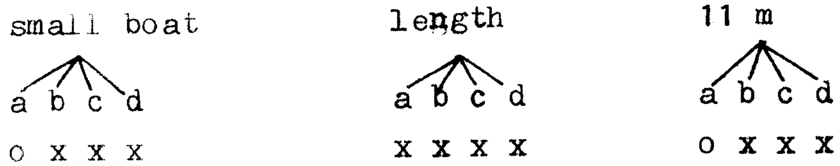
is solved.
Due to

$$K_{r'}^{(1)}(TR_2^{(1)}) > 0$$

Then

$$K_{r'}^{(3)}(R_2^{(3)}) < 0$$

So we must use a separation-reunion transformation [1].
First the matter element $R_2^{(1)}$



is analyzed.

It is shown that we can use the combination of the transformation. Namely, both of the boats are discharged together at the same time.

The matter element (Small boat, discharging time, 30 hours) will be turned into total of discharging time 15 hours. So

$$K_{r'}^{(1)}\left(TR_2^{(1)}\right) > -1$$

$$K_{r'}^{(3)}\left(TR_2^{(3)}\right) > 0$$

From above statement, we get two methods for solving this problem.

1. Two large ships will be discharged in different times.

At the beginning of the high tide, one of the large ships will be discharged. After an hour the ship must sail out from the warf. The another ship will be discharged off all goods and then, the first ship will be discharged again. Since it finishing discharging a part of goods. It can be sail into the wharf freely at ebb tide.

2. Let two small ship sail into the wharf and be discharged at the same time at ebb tide.

The problem is solved.

Reference Article

[1] Cai Wen "Extension Set and Non-compatible problems". « Science Exploration » 1 (1983).

[2] Cai Wen "Essentials of the matter element analysis". « Journal of Artifical intelligence » 2 (1983).