

## On possibility theory

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The possibility theory unifying both formal and intuitive approaches must be built upon historical notions of possibility, which are developed in philosophy and science.

There are five directions of constructing such a possibility theory: (1) historical analysis, (2) formal definition of possibility, (3) intuitive notions of possibility in science and art, (4) methodological problems, (5) applications.

(1) The historical analysis can be started from the notion of possibility ("dynamis") by Aristotle. In his *Metaphysics* V/12 the possibility is defined as "the start of motion or change, which takes place in another thing". Later, G.W. Leibniz has shown, that a priori possibility allows a breakdown of an event into elementary notions with known possibility. I. Kant takes as possible that, what corresponds to formal conditions of experience. He describes three modalities : reality, possibility and necessity. G.W.F. Hegel more exactly derives the formal, real and absolute possibility. In Hegelian sense, the necessity is a relation between reality and possibility. W.M. Rosenthal explains possibility as an ability of matter to obtain different forms during the process of motion - reality is the materialized possibility. So, the real and abstract (formal) possibility are distinguished.

(2) Following the approach of Leibniz, a possibility distribution for a variable  $X$  can be defined as

$$\bigcup_{u \in U} \text{Poss} (X=u) ,$$

where  $\text{Poss}(X=u)$  is the level of easiness, with which the variable  $X$  attains the value  $u$  from an universe  $U$ . From known possibility distributions of elementary events the possibility of an event ( $Q$ -event, second-level-event) can be derived. Borisov and Krumberg call the possibility of an  $Q$ -event "elementary possibility" and say, that the elementary possibility denotes the ability of object  $Q$  to be an ideal system  $S$  in sense of the notion  $Y$ . So, the possibility distribution, formally specified by a fuzzy restriction, is the fundamental for the formal theory of possibility.

(3) In the intuitive sense the "possibilistic approach" is opposite to the "probabilistic approach". This means, that in the probabilistic approach we consider old, already materialized and known trends, but in the possibility approach we search for new, original and surprising outcomes. Such different points of view are presented in special sciences. E.g. in economics a system of reserves for purposes of further growth is necessary. In sociology a plurality of development potentials is analysed in conditions of a region or a social group. In psychology abilities of an individual, its talent, gifts and aspirations are precisely stated before they are realized. In planning and control the probabilistic approach leads to extrapolation of old time series, the potential approach enlarges the possibility of new goals, programs and parameters. In the system theory the probabilistic approach is related with a scientific paradigm, analysis and deduction, the potential approach with the system paradigm, synthesis and design of more complex systems.

(4) The possibility theory represents a methodology for studies in complex systems. System problems as identification, modelling or implementation should be solved from the point of view of possibility theory as fuzzy systems identification, modelling or implementation. A great number of known techniques using fuzzy sets belong to this possibility methodology. The central position in future can reach the theory of fuzzy categories as appropriate models for human and social systems. Further the

fuzzy logic, as a branch of modal and many-valued logics, is useful for approximate and linguistic modelling and decision-making. Also methods of multicriteria fuzzy decision-making and information retrieval make a part of possibility methodology.

(5) Applications of possibility theory are more common than its formal notions constructed by means of fuzzy sets. The creative approach to the problem solving ("the art of problem solving") is older than the modern possibility theory. But in context of very complex systems of our world the formulation and solution of highly sophisticated tasks needs for new tools. Theoretical and methodological basis represented by the possibility theory opens new ways for applications in production and leisure. For example, tasks using fuzzy information and fuzzy time allow to consider relativity in human affairs, e.g. a different value of each portion of messages or of each hour in our individual life.

So, the modern possibility theory must be founded on certain philosophy and "Weltanschauung". There begins a general definition of possibility. There are also important issues in modal and many-valued logics, which are useful for the fuzzy logic as "logic of possibility". Further, formal notion of possibility, based on possibility distribution, fuzzy restriction and fuzzy sets, has to be developed. Interpretation of formal notions in the context of special sciences, as economics, sociology, history or ecology, enlarges the scope of intuitive possibility theory. Many techniques in fuzzy modelling and decision-making form together a system methodology (a new phase of the system analysis). Finally, applications of the possibility theory lay in the area of creative problem solving, where the "New" cannot be derived from an "Old", but needs for different ways of comparison, classification and questioning.