A note on linguistic hedges and the hedge "very"

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1. INTRODUCTION

The linguistic hedges are very important in a fuzzy semantics which is a model of the semantics of natural language. They form a subclass of adverbials which make the meaning of a verbal expression more exact. From the point of fuzzy sets theory they modify the membership function of a fuzzy set. Another property is that they induce ordering of the universe U. We shall, moreover, suppose that this ordering is linear and that the cardinality of U is at most that of continuum.

In this paper, we summarize shortly the present stage of linguistic hedges in the fuzzy sets theory. In the section 3 an empirical model of the hedge "very" is proposed and shortly discussed.

2. LINGUISTIC HEDGES IN THE FUZZY SETS THEORY

Let $\mathcal{A}$ be a verbal expression. Its meaning is a fuzzy set $A \underset{\sim}{\subseteq} U$. We shall suppose for simplicity that $U \subseteq$ Real numbers.

Let $\mathfrak{m}$ be a linguistic hedge. It is adjoined an associated function

$$Asf_{\mathfrak{m}} : L \rightarrow L$$

where L is the set of grades of membership (usually $L = \langle 0, 1 \rangle$). Then the meaning of $\mathfrak{m}\mathcal{A}$ is

$$M(\mathfrak{m}\mathcal{A}) = Asf_{\mathfrak{m}} \circ A$$

where $A = M(\mathcal{A})$ is the meaning of $\mathcal{A}$.

The following condition must hold for arbitrary associated function:

(PO) $((\alpha \to \beta) \wedge (\beta \to \alpha))^k \leqq (Asf_m(\alpha) \to Asf_m(\beta)) \wedge (Asf_m(\beta) \to Asf_m(\alpha))$

for every $\alpha, \beta \in L$ and natural number k. It states that "approximately equal" arguments make the values of associated function "approximately equal", too.

Four basic functions from L^L are introduced ($L = \langle 0, 1 \rangle$):

$$CON(\alpha) = \alpha^2$$

$$DIL(\alpha) = \neg CON(\neg\alpha) = 2\alpha - \alpha^2 \qquad (\neg\alpha = 1 - \alpha)$$

$$INT(\alpha) = ((\neg CON(\alpha) \to CON(\alpha)) \wedge \alpha) \vee (\neg CON(\neg\alpha))^2 = \begin{cases} 2\alpha^2 & \alpha \in \langle 0, 0.5 \rangle \\ 1 - 2(1-\alpha)^2 & \alpha \in \langle 0.5, 1 \rangle \end{cases}$$

$$NORM(f(\alpha)) = \frac{f(\alpha)}{\bigvee_{\alpha \in L} f(\alpha)} \qquad f: L \to L$$

Using these functions, we construct associated functions of the following hedges:

very	$Asf_{very}(\alpha) = CON(\alpha) = \alpha^2$
more or less	$Asf_{more\ or\ less}(\alpha) = DIL(\alpha)$
highly	$Asf_{highly}(\alpha) = \alpha^3$
roughly	$Asf_{roughly}(\alpha) = DIL(DIL(\alpha)) = -\alpha^4 + 4\alpha^3 - 6\alpha^2 + 4\alpha$
rather	$Asf_{rather}(\alpha) = INT(CON(\alpha)) = (2\alpha^4 \wedge \alpha^2) \vee (-2\alpha^4 + 4\alpha^2 - 1)$
slightly	$Asf_{slightly}(\alpha) = NORM(\alpha \wedge \neg CON(\alpha)) = \dfrac{2(\alpha \wedge (1 - \alpha^2))}{\sqrt{5} - 1}$

All the associated functions fullfil the condition (PO).

3. EMPIRICAL MODEL OF THE HEDGE "VERY"

The linguistic hedge "very" is the most freequently used hedge in applications of fuzzy sets. The following model is based on empirical data [1].

Lakoff [2] pointed out that associated function of the hedge "very" does not depend on the grades of membership only but also on the elements of the universe U.

Given a sufficiently rich set $\mathcal{T}$ of verbal expressions. Using the ordering of U they can be characterized as follows:

a) There exest couples of expressions $\mathcal{A}^{+}, \mathcal{A}^{-} \in \mathcal{T}$ which we shall call <u>polar couples</u>. A typical example are e.g. "small, good", "young, old" etc.

b) The other expressions are not members of any polar couple and we shall call them <u>non-polar</u>. A typical example is "average".

The characterizing points m, s, $v \in U$ should be determined fulfiling the conditions

a) $m \lneq s \lneq v$.

b) For any $\mathcal{A} \in \mathcal{T}$

$$\mathrm{Supp}(A) \subseteq \langle m, v \rangle$$

where $A = M(\mathcal{A})$ and Supp(A) is support of A.

c) The element s is in some sense among all the polar couples $\mathcal{A}^{+}, \mathcal{A}^{-} \in \mathcal{T}$. It will be called <u>semantical center</u> of U.

The elements m, s, v are determined ambiguously. Their exact position is subjective dependent on concrete situation.

Suppose that all the α-cuts A_{α} of A for any $\mathcal{A} \in \mathcal{T}$ are intervals in U and let $\mathrm{Ker}(A) = \{x;\ Ax = 1\}$ be the kernel of A.

a) $\mathrm{Ker}(A) \subseteq (s, v\rangle$ then $\mathcal{A}$ is polar expression (member of some polar couple) denoted by $\mathcal{A}^{+}$ and its meaning by A^{+}.

b) $\mathrm{Ker}(A) \subseteq \langle m, s)$ then $\mathcal{A}$ is polar expression denoted by $\mathcal{A}^{-}$ and its meaning by A^{-} .

c) $s \in \mathrm{Ker}(A)$ then $\mathcal{A}$ is non-polar expression denoted by

$\mathcal{A}^0$ with meaning A^0.

The hedge "very" modifies membership function in two ways:

a) It holds for width of all α-cuts (intervals of real numbers by presumption)

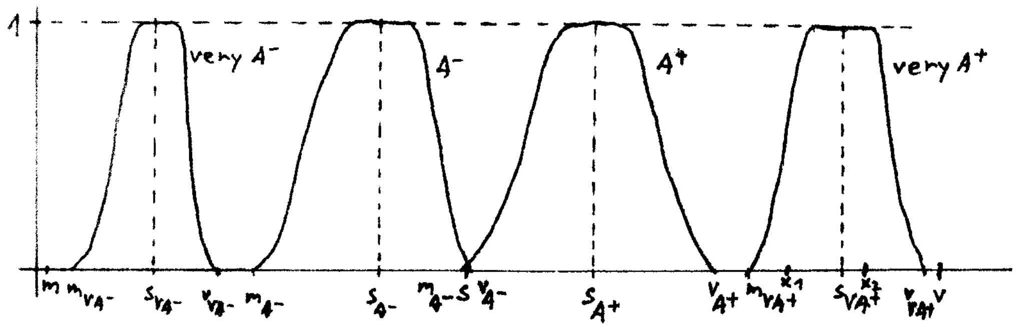
$$|(\text{very } A)_\alpha| \lneqq |A_\alpha|$$

where very A = M(very $\mathcal{A}$) .

b) The membership function of the fuzzy set $A = M(\mathcal{A})$ is

- moved to the right (towards v) if $\mathcal{A}$ is polar expression $\mathcal{A}^+$,
- moved to the left (towards m) if $\mathcal{A}$ is polar expression $\mathcal{A}^-$,
- not moved if $\mathcal{A}$ is non-polar expression $\mathcal{A}^0$.

The situation can be illustrated on this picture



We demand to be

$$\text{Ker}(\text{very } A^+) \cap \langle x_2, v \rangle \neq \emptyset$$

and $|\langle x_1, v_{A^+}| = |\langle x_2, x_1 \rangle| = |\langle v, x_2 \rangle|$ and similarly for the expression $\mathcal{A}^-$. Then the meaning of very $\mathcal{A}$ is the fuzzy set (very A) with the membership function

$$(\text{very } A)(x) = \begin{cases} A(x - M(A(x - \Delta_A)).\Delta_A) & \text{if } A = A^+, \text{ or} \\ & A = A^0 \text{ and } x \lneqq s \\ A(x + M(A(x + \Delta_A)).\Delta_A) & \text{if } a = A^-, \text{ or} \\ & A = A^0 \text{ and } x \geqq s \end{cases}$$

where M(y) is a decreasing function of y and Δ_A is a number

$$ {}_A = \begin{cases} \dfrac{2v + m_A}{3} - s_A & \text{for } A = A^+ \\ s_A - \dfrac{2m + v_A}{3} & \text{for } A = A^- \end{cases} $$

$$ \Delta_A < \frac{|Ker(A)|}{2} \qquad \text{for } A = A^0 $$

The function $M(y)$ can be e.g. linear function

$$ M(y) = q - py $$

where p, q are numbers selected dependingly on the context and other factors. Probable values of p, q are $p \approx 0.2 \div 0.4$, $q \approx 0.9 \div 1.4$.

4. DISCUSSION

The model of the hedge "very" presented here differs from that in the section 2 by non-exactness of the associated function. Practical investigation shows that there really exist some general laws in the natural language semantics which can be quantified. They are, however, non exact which must be respected also in the fuzzy sets theory. The concrete meaning of the verbal expression strongly depends on the given situation, context, the knowledge of the speaker and listener, their mood and temperament, and probably other factors. There is still much work which has to be done in the fuzzy semantics.

REFERENCES

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[2] Lakoff, G.: Hedges: a study in meaning criteria and the logic of fuzzy concepts. J. Philos. Logic 2, 1973, 458-508.

[3] Zadeh, L.A.: Quantitative Fuzzy Semantics. Inf. Sci. 3, 1973, 159 - 176.