FUZZY ADAPTIVE MODEL OF DECISION-MAKING PROCESS

A.F. BLISHUN

Institute of Automated Management
Systems of The Ministry of Fishing
Industry, Moscow, USSR

The management of complex systems often needs the non-formal reasoning on the base of intuitional subjective knowledge representation. During the automation of the management such reasoning can't be described in adequate quantitative form. In particular, there are many problems of modelling of decision-making process with the following characteristics: presence of decision-maker, importance of subjective factors, dynamics of subjective factors.

The solution of problems mentioned above requires suitable mathematical models which are able to generalize fuzzy information to adopt to new situation. These models are capable of improving their performance during operation.

In this paper an adaptive model of this type is suggested.

The decision making process may be described as selection of a particular alternative from a set of possible ones. When analysing the situation possible alternatives can be mapped in attribute space or semantic space. In semantic space an alternative displays the attribute with certain degree. This degree is measured by means of scale of measurements. Well known, however, that in the most of cases human's decision are vaguely or imprecisely defined so that the grade of attribute is measured by expert very approximately.

Therefore, a method of scaling must testify the following conditions [1] :

it should allow for a moderate uncertainty in judgments about grades of attribute without changing the judgment value on the scale;

strong changes of the judgment should be reflected in the appropriate variations up and down the scale;

the results of the model should not change drastically by making small changes in the judgment values.

The fuzzy scale satisfies the above conditions.

Definition 1. Let U be a nonempty set of empirical objects, R_i (i=1, ...,n) a set of relations over U, $L \subseteq R$ (the set of real numbers), T a set of fuzzy subset of L, S_i (i=1,...,n) a set of relations over T.
Let R_i and S_i be k_i-place relations.
Then a mapping $M: U \to T$ is a fuzzy scale if

$$R_i(u_1,\dots,u_{k_i})=S_i(m(u_1),\dots,m(u_{k_i}))$$

for all $i \in I=\{1,\dots,n\}$ and $(u_1,\dots,u_{k_i}) \in U^{k_i}$.

It is clear that from relations between judgment values on fuzzy scale we are able to draw a conclusion about empirical relations between empirical objects as follows: empirical objects $u_1,\dots,u_{k_i}$ are in relation R if and only if corresponding values $m(u_1),\dots,m(u_{k_i})$ on fuzzy scale are in relation S.

In oder to fit the above problems into a certain mathematical model it seems reasonable to satisfy the following conditions concerning the fuzzy scale.

Hypothesis I. If an attribute is characterised by measurable feature, then fuzzy scale must ensure the monotonous mapping between values of feature and semantic space.

Hypothesis II. In the case of a finite number of elements of fuzzy scale neighbouring values give equal change of grade of attribute, i.e. the "physical" distances of each two adjacent elements of fuzzy scale in semantic space are equal [2,3,4] .

Dependly on situation and personal perceptions of experts the grade of the same attribute can be eveluated differently.

When real values of features are identified with semantic image we have to find adequate form of mapping between feature space (U) and semantic space (P). The mapping has to preserve a relative place of objects in semantic space.

A natural way to formalize the concept of fuzzy mapping is via the concept of fuzzy property [2] .

Definition 2. A property p defined on an object u is a function $p: U \to [0,1]$ such that $p(u)>0$ iff u has the property p.

Definitions 1 and 2 follow by the fact that grades of attribute correspond an ordered set $P=\{p(u): u \in U\}$. Consequently, each element of fuzzy scale corresponds to a fuzzy subset of P. According to Definition 1 the attribute can be interpreted as a linguistic variable whose values are the labels of fuzzy subsets on P. Figure 1 shows the correspondence of judgements about grades of attribute in semantic space and feature space.

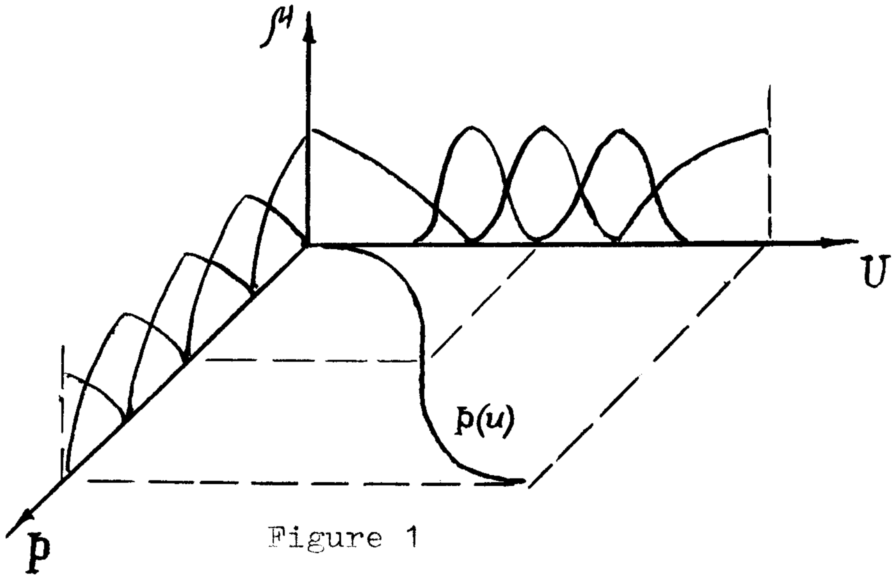


Figure 1

When fuzzy scales are used reception of supplementary information or change of personal entails their adjustment by means of either displacement of fuzzy scale elements or change of mapping p(.).

This paper has deal with the first case only.

FORMULATION OF THE PROBLEM

Let a rational decision choice be realized by means of a collective of strictly unformed criteria, which can't be described in adequate analytical form. In this case preferences are based on the following linguistic statement:

"if values of features $u_1,\dots,u_n$ which characterize an alternative u^i are evaluated by terms $t_{1i},\dots,t_{ni}$, then u^i satisfies the j-th criterion with value $t_{n+j,i}$ ".

Every alternative can be described in terms of composit linguistic variable

$$L=(L_1,\dots,L_n,\dots,L_{n+k},L_{n+k+1}),$$

where L_i is an unary linguistic variable with a term-set T_i, with an universal set U_i, and with a basic variable u_i, $t_{il}\in T_i\subseteq F(U_i)$.

Values from a term-set $t_{il}\in T_i$, $n+1\le i\le k+n+1$, point out a grade of criterion satisfaction. T_i consists of linguistic values of linguistic variable.

A set of values of L is separated on two subsets:

$$M_1=\left\{(t_{1i},\dots,t_{ni},t_{n+j,i})\,\middle|\,n+1\le j\le n+k\right\}_{i=1}^{i=m_1},$$

$$M_2=\left\{(t_{n+1,i},\dots,t_{n+k+1,i})\right\}_{i=1}^{i=m_2}.$$

M_1 characterizies local criteria and M_2 characterizies global criterion. $M=M_1\cup M_2$ consists of a priori information which is obtained from expert.

Each $t, t\in M$, can be presented as a fuzzy point in a subspace of D, where D is presented as the Cartesian product of feature space and criteria space [5]. In reality, any value of feature u_i has correspondent degree of property p_i. Hence, u^i defines the point $(p_1(u_1),\dots,p_n(u_n))$ in the semantic space $P_1 \dots P_n$ and if t^i M, t^i is defines the fuzzy point $t^i(p(.))$.

For an alternative u^i the grade of global criterion satisfaction $w(u^i)$ is computed as:

$$w(u^i)=\bigcup_{t^r\in M_2}\Big(\bigcup_{t^l\in M_1}u^i\circ t^l\Big)\circ t^r \qquad (1)$$

Assume that a set of alternatives $R'=\{u^i\}=\{(u_1^i,\dots,u_n^i)\}$ is given to the decision-maker. He chooses a set of alternatives R'' from the given set R' : $R''\subseteq R'$.

Using the preference $u^i\succ u^j$, $u^i\in R''$, $u^j\in R'\setminus R''$, it is necessary to replace M by some M' which confirms the above preference in a following way:

$w'(u^i)\succ w'(u^j)$ for all $u^i\in R''$, $u^j\in R'\setminus R''$.

In order to obtain a more precise description of fuzzy adaptive algorithm [6] we have introduced following definitions.

Definition 3. Let $M=\{t^i=(t^i_1,\dots,t^i_{n+k+1})\}$, $t^i_j\in T_j\cup 0$, $t^1,t^2\in M$, $t^1=(t^1_1,\dots,t^1_n,t^1_{n+j_1})$, $j_1\leq k$, $t^2=(t^2_1,\dots,t^2_{n+j_2})$, $j_2\leq k$. Let Z be the truth-value fanction. We say that t^2 implies t^1 (denoted $t^1\leq t^2$) if the formula

$$\mathop{\&}_{i=1}^{n}\left(Z(t^1_i=t^2_i)\vee \neg Z(t^1_i\neq 0)\ \&\ Z(t^2_i\neq 0)\ \right)$$

is valid.

Definition 4. A n-tuple of numbers $n^f=(n^f_1,\dots,n^f_n)$ is called the mask of n-tuple $v^{ij}=(v^{ij}_1,\dots,v^{ij}_n)$ if n^f_l defined as follows:

$$n^f_l=\begin{cases}1, & \text{if } v^{ij}_l>0.5+\varepsilon,\\ -1, & \text{if } 0<v^{ij}_l<0.5-\varepsilon,\\ 0, & \text{otherwise},\end{cases}\qquad 0<\varepsilon<0.5\ .$$

Definition 5. Let $u^i=(u^i_1,\dots,u^i_n)$ and $u^j=(u^j_1,\dots,u^j_n)$ be two n-tuples of values of associated features $u_1,\dots,u_n$. The l-th feature is called

a) an essential feature, if $p(u^i_l)\neq p(u^j_l)$;

b) an ε-essential feature, if $|p(u^i_l)-p(u^j_l)|>\varepsilon$.

Definition 6. A logical feature of the table $K=\{v^{ij}=(v^{ij}_1,\dots,v^{ij}_n)\}$ is defined as follows:

$$\Phi_{f_1,f_2}(v^{ij})=\bigwedge_{v^{ij}\in K} F(\ D_{n^{f_1}}(v^{ij}),D_{n^{f_2}}(v^{ij})\ ),$$

where

$$D_{n^f}(v^{ij})=v^{ij\ n^f_1}_1\ \circledvee\ \dots\ \circledvee\ v^{ij\ n^f_n}_n\ ,$$

$n^f\in N$, $N=\{n^f\}$;

$\circledvee = \vee$ or $\wedge$,

$$v^{ij\,n^f_l}_l=\begin{cases}1-v^{ij}_l, & \text{if } n^f_l<0,\\ v^{ij}_l, & \text{if } n^f_l>0,\\ 1, & \text{if } n^f_l=0,\end{cases}$$

F - a logical functoin of two variables.

ALGORITHM OF SOLUTION

Block diagram of fuzzy adaptive algorithm is shown in Figure 2. Let us describe this algorithm in detailed.

I. Compression of M.

In this step all pairs (t^i, t^j) are analysed. If t^i implies t^j then t^i is deleted.

II. Construction of tables for learning.

Let us consider two classes of pairs of alternatives:

$$K_1=\{(u^i,u^j) : u^i \in R'', u^j \in R'\setminus R''\},$$

$$K_2=\{(u^j,u^i) : u^i \in R'', u^j \in R'\setminus R''\}.$$

Tables for learning K_1^T, K_2^T are formulated when K_1, K_2 have the following properties:

1) $K_1 \neq \emptyset$, $K_2 \neq \emptyset$;

2) $\forall i\ \exists j: \ w(u^j) \propto w(u^i) \quad u^i \in R'', \ u^j \in R'\setminus R''$.

Each pair of alternatives $(u^i,u^j) \in K_1 \cup K_2$ defines the row of table for learning

$$v^{ij}=(v_1^{ij},\ldots,v_n^{ij}),$$

where

$$v_l^{ij}=\begin{cases}-1, & \text{if value of feature } u_l \text{ is not defined,}\\ \dfrac{1+p_l(u_l^i)-p_l(u_l^j)}{2}, & \text{otherwise.}\end{cases}$$

Then we have

$$K_1^T=\{v^{ij}=(v_1^{ij},\ldots,v_n^{ij})\},$$

$$K_2^T=\{v^{ji}=(v_1^{ji},\ldots,v_n^{ji})\}.$$

III. Formation of masks.

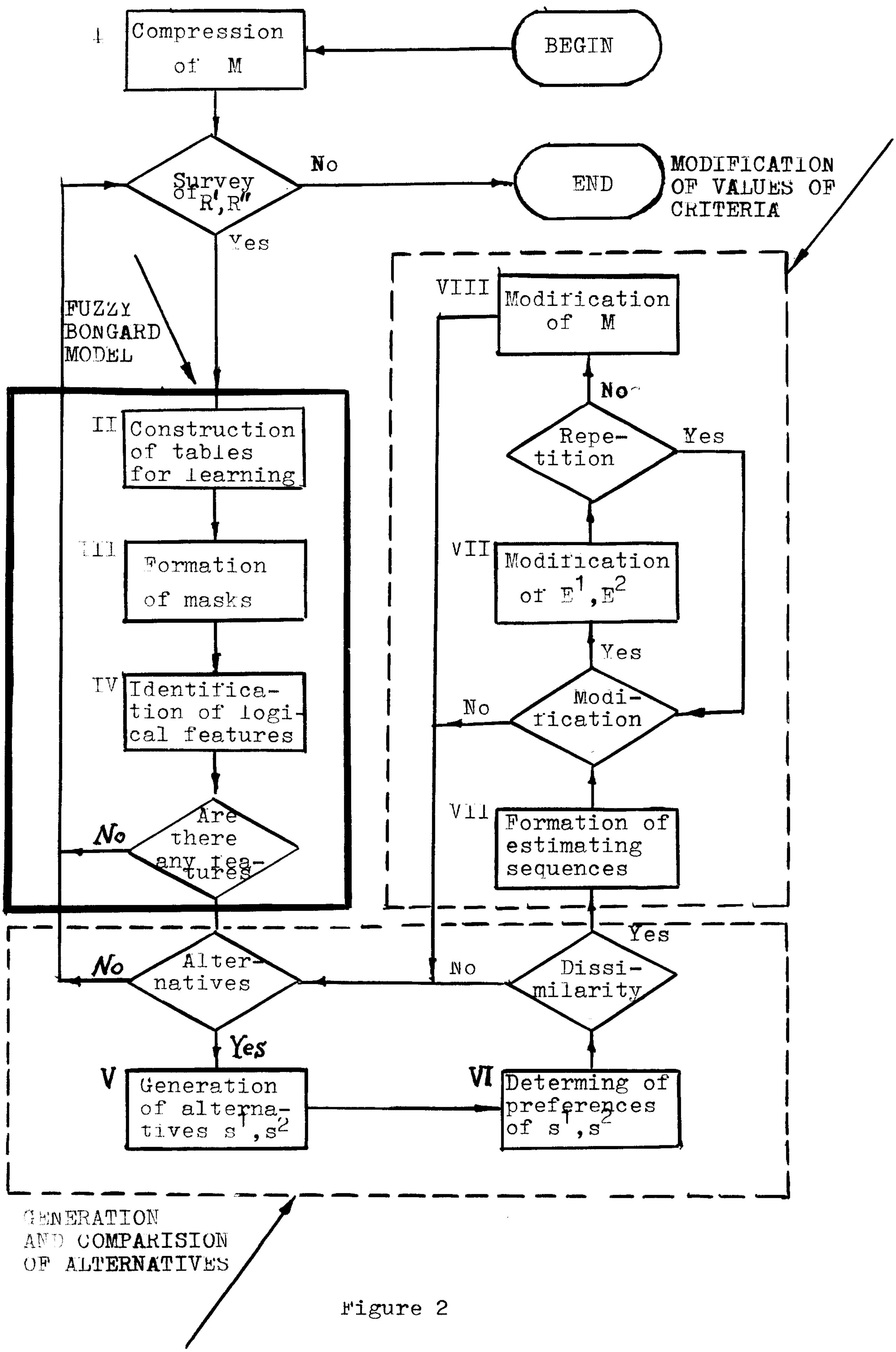
The set of all masks $N=\{n^f\}$ over $K_1^T \cup K_2^T$ define possible index sets of 2ε-essential features.

IV. Identification of useful logical features.

Each table K_i^T $(i=1,2)$ is separated into working and verifying tables K_{iw}^T, K_{iv}^T :

$$K_i^T=K_{iw}^T \cup K_{iv}^T, \quad K_{iw}^T \cap K_{iv}^T = \emptyset; \quad K_{iw}^T, K_{iv}^T \neq \emptyset.$$

Let C_w^l, C_v^l $(l=1,2)$ be the calculated values of logical features over K_{iw}^T, K_{iv}^T, ε_1 maximal admissible distinction of



I
Compression of M
BEGIN
Survey of R',R''
No
END
MODIFICATION OF VALUES OF CRITERIA
Yes
FUZZY BONGARD MODEL
VIII
Modification of M
No
II
Construction of tables for learning
Repe- tition
Yes
III
Formation of masks
VII
Modification of E1,E2
Yes
IV
Identification of logical features
No
Modi- fication
VII
Formation of estimating sequences
No
Are there any features
Yes
No
Alter- natives
No
Dissi- milarity
Yes
V
Generation of alternatives s1,s2
VI
Determing of preferences of s1,s2
GENERATION AND COMPARISION OF ALTERNATIVES

Figure 2

K^T_{iw} and K^T_{iv} (i=1,2), ε_2 minimal admissible distinction of K^T_1, K^T_2. Then the value of logical feature over K^T_l is given as follows:

$$\mu_l = C^l_w \wedge C^l_v \ .$$

Definition 7. A logical feature is called an useful logical feature, if

$$1)\ \Delta\mu_l < \varepsilon_1,\ l=1,2;$$

$$2)\ \Delta\mu_{12} > \varepsilon_2\ ,$$

where

$$\Delta\mu_l = C^l_w \vee C^l_v - C^l_w \wedge C^l_v\ ,$$

$$\Delta\mu_{12} = \mu_1 \vee \mu_2 - \mu_1 \wedge \mu_2\ .$$

Definition 8. Useful logical features $\Phi_1, \dots, \Phi_r$ form a basis of features, if either $\mu^+_1 < \mu^-_2$ or $\mu^+_2 < \mu^-_1$,

where

$$\mu^+_l = \bigvee_{v \in K^T_l} \bigwedge_{j=1}^{r} \Phi_j(v), \quad \mu^-_l = \bigwedge_{v \in K^T_l} \bigwedge_{j=1}^{r} \Phi_j(v); \quad l=1,2.$$

Figure 3 illustrates the definition of basis of features.

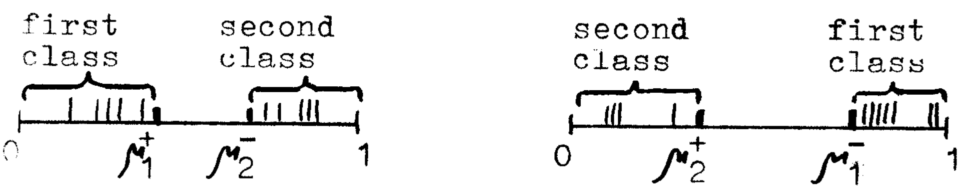


Figure 3

If the number of useful logical features in some basis is minimal, then the basis is minimal basis.

On this step all useful logical features are identified.

V. Generation of alternatives.

If a preference relation obtained by means of (1) does not coincide with a preference relation obtained by means of useful logical features then modification of M is necessary. The above preferences is defined over set of alternatives which are generated by the following way.

Each set $t \in M$ can be represented as a strict point in the semantic space. An expedience of this representation is confirmed by the second hypothesis and unimodal membership function $\mu_t(p)$. The above simplification allows to use in the algorithm only the operations on fuzzy sets of type 1. The algorithm which can deal with fuzzy sets of type 2, has been considered in [7].

Elements of T_i, $|T_i| = g_i$, are evenly distributed over the interval $[1, g_i]$. In this case 1 corresponds to the best value from T_i, g_i corresponds to the worst value from T_i (see Fig. 4). Elements of T_i can be represented as a strict point in the semantic space.

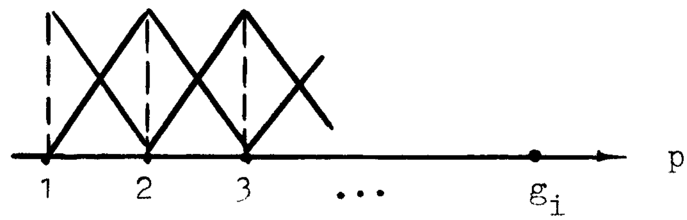


Figure 4

We obtain the following condition of generation of pairs of alternatives (s^1, s^2), $s^1 = (s_1^1, \dots, s_n^1)$, $s^2 = (s_1^2, \dots, s_n^2)$, $s_i^j \in [1, g_i]$:

$$\max_i \left| s_i^1 - s_i^2 \right| = 1 .$$

VI. Determing of preference of generated alternatives s^1, s^2.

For s^1, s^2 the set of comparison values is computed as

$$s^{12} = (s_1^{12}, \dots, s_n^{12}),$$

where

$$s_i^{12} = \frac{1 + (s_i^1 - s_i^2)/(g_i - 1)}{2} .$$

The degree of membership of s^{12} in l-th class is given by equation

$$\mu_l(s^{12}) = \bigwedge_{i=1}^{r} \left(1 - \left| \Phi_i(s^{12}) - \mu_l^i \right| \right),$$

where

$$\mu_l^i = \bigwedge_{v \in K_l^T} \Phi_i(v), \quad l = 1, 2.$$

If $\mu_1(s^{12}) > \mu_2(s^{12})$, then $s^1 \succ s^2$. If $\mu_1(s^{12}) < \mu_2(s^{12})$, then $s^1 \prec s^2$.

VII. Formation and modification of estimating sequences of s^1, s^2.

Definition 9. Let $s=(s_1,\ldots,s_n)$ be an alternative. A sequence of numbers $E=(e_1,\ldots,e_{n+k+1})$ is called an estimating sequence of s if:

1) $e_i=s_i$ for $i=1, \ldots, n$;

2) $e_i=\max\limits_{\substack{t=(t_1,\ldots,t_{n+k+i})\in M\\ t:\ s_j=t_j,\ j=1,\ldots,n}} t_i$ for $i=n+1,\ldots,n+k$

3) $e_{n+k+1}=t_{n+k+1}$,

t_{n+k+1}: a) $t=(s_1,\ldots,s_n,s'_{n+1},\ldots,s'_{n+k},t_{n+k+1})\in M$;

b) $|s'_i-e_i|<0.5$, $i=n+1,\ldots,n+k$.

Let E^1,E^2 be estimating sequence of numbers of s^1,s^2 and ε_3 be a correction step value Δ of criterion guesses for $t\in M$, then

$$\Delta=\begin{cases}+\varepsilon_3, & \text{if } s^1_{n+k+1}\leq s^2_{n+k+1} \text{ and } \mu_1(s^{12})<\mu_2(s^{12}),\\ -\varepsilon_3, & \text{if } s^1_{n+k+1}\geq s^2_{n+k+1} \text{ and } \mu_1(s^{12})>\mu_2(s^{12}),\\ 0, & \text{otherwise.}\end{cases}$$

From here we conclude that

$\Delta>0$, if $E^1\succsim E^2$ and $s^1\prec s^2$,

$\Delta<0$, if $E^1\precsim E^2$ and $s^1\succ s^2$.

Criterion estimates are corrected only in E^1,E^2.

For each local criterion a number δ_i is computed:

$$\delta_i=\begin{cases}1, & \text{if } \forall j\in I_r \quad e^1_j>e^2_j \text{ and } e^1_{n+i}\leq e^2_{n+i},\\ -1, & \text{if } \forall j\in I_r \quad e^1_j<e^2_j \text{ and } e^1_{n+i}\geq e^2_{n+i},\\ 0, & \text{othewise}.\end{cases}$$

The set $I=\{j_1,\ldots,j_n\}$ contains numbers of features which characterize the i-th local criterion, that is

$$I=\{j\,/\,\exists\ t=(t_1,\ldots,t_{n+i})\in M:\ t_j\neq 0 \text{ and } t_{n+i}\neq\emptyset,\ j\leq n\}.$$

The modification of local criteria is given by

$$e^1_{n+i}=\min\{g_{n+i},\ \max(1,e^1_{n+i}+\delta_i|\Delta|(g_{n+i}-1))\},$$

$$e^2_{n+i}=\min\{g_{n+i},\ \max(1,e^2_{n+i}-\delta_i|\Delta|(g_{n+i}-1))\}.$$

The modification of global criterion is given by

$$e^1_{n+k+1}=\min\left\{g_{n+k+1},\ \max(1,e^1_{n+k+1}+\Delta(g_{n+k+1}-1))\right\},$$

$$e^2_{n+k+1}=\min\left\{g_{n+k+1},\ \max(1,e^2_{n+k+1}-\Delta(g_{n+k+1}-1))\right\}.$$

VIII. Modification of M.

Let N^1_e, N^2_e be a set of all numbers of lists from M. Set M used in the constiuction of $E^1, E^2, N^{12}_e=N^1_e \setminus N^2_e$, $N^{21}_e=N^2_e \setminus N^1_e$. We substitute M by $\hat{M}$,

$$\hat{M}= \hat{p}^i=(\hat{p}^i_1,\ldots,\hat{p}^i_{n+k+1}),$$

where

$$\hat{p}^i_j=\begin{cases} p^i_j, & \text{if } i \notin N^{12}_e \cup N^{21}_e, \\ 0, & \text{if } p^i_j=0, \\ e^1_j, & \text{if } i\in N^{12}_e, \\ e^2_j, & \text{if } i\in N^{21}_e. \end{cases}$$

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