AN ATTEMPT OF A FUZZY APPROACH TO VOLTERRA SYSTEMS

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Investigations of rough behaviour and robustness of Volterra systems are of special interest. For this porpose an attempt is made to describe Volterra systems like

$$\frac{\dot{x}_i}{x_i} = \sum_j G_{ij} x_j \quad , \qquad i=1(1)n, \quad j=1,(1)n+1 \qquad (1)$$

$$x_{n+1} = \text{const.}$$

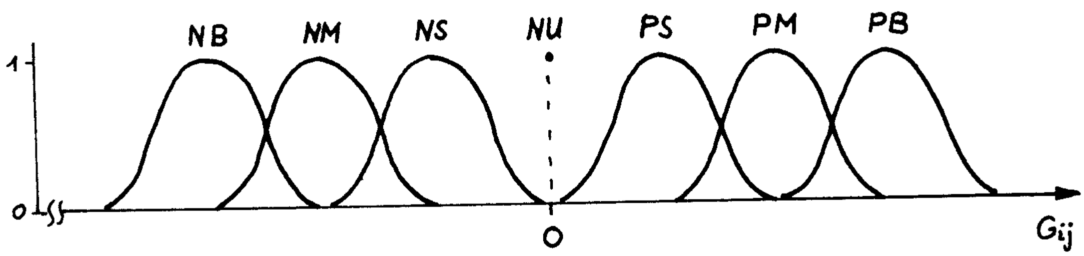
in a fuzzy manner.

First of all, both time and ranges of the x_i and the parameters G_{ij} are discretized.
Discretization of time is obtained by applying Euler foreward differences

$$x_i(k+1) = x_i(k) + \Delta x_i(k)$$
$$x_i(k) + h\cdot\dot{x}_i(k) = x_i(k)[1 + h\frac{\dot{x}_i(k)}{x_i(k)}] \qquad (2)$$

The ordered classes A,B,...,G,H are used for discretizing x_i whereas A means the fuzzy set 'very small' and H stands for 'very large'.

Fuzzy sets are also defined on the set of parameters G_{ij}. Here is a schematic representation of it:



The fuzzy set NU is a (fuzzy) singleton with $NU(G_{ij})=1$ if $G_{ij}=0$ and $NU(G_{ij})=0$ if $G_{ij}\neq 0$.
Hence, a non-interaction between x_j and x_i (i.e., $G_{ij}=0$ in the original system) is preserved in the fuzzy model. All fuzzy sets introduced above have the following meaning NB (PB) 'negative (positive) big', NM (PM) 'negative (positive) medium', NS (PS) 'negative (positive) small', and NU 'uncoupled'.

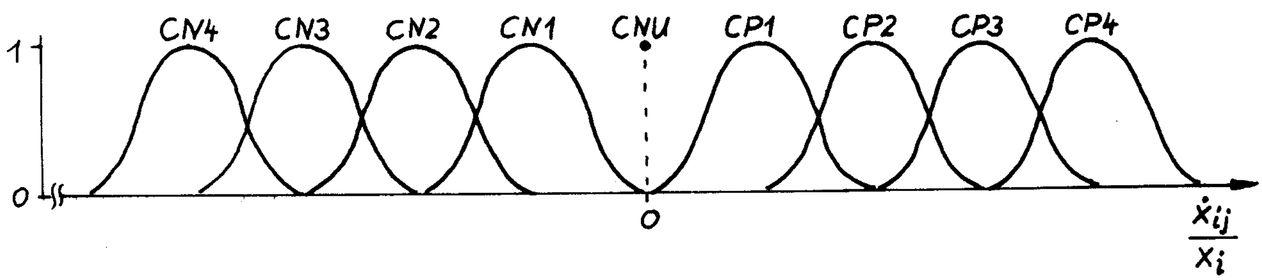
Considering the time k the known values of fuzzy classes of $x_j(k)$ have to be combined with the fuzzy values of G_{ij} for all j in a manner explained later. This leads to the relative growth rate $[\dot{x}_i/x_i](k)$.
The partial relative growth rate caused by x_j via G_{ij} is denoted by

$$\frac{\dot{x}_{ij}}{x_i} = G_{ij}x_j$$

Then, (1) is read

$$\frac{\dot{x}_i}{x_i} = \sum_j \frac{\dot{x}_{ij}}{x_i}$$

Now, fuzzy sets for the relative growth rates are defined on the set of $\dot{x}_{ij}/x_i$ (see also the fuzzy parameters for G_{ij} given above):



The interrelationship between the fuzzy classes of x_i, the fuzzy values of G_{ij}, and the fuzzy partial relative growth rates $\dot{x}_{ij}/x_i$ is seized by the fuzzy algorithm GX whose underlying operational table is shown in Table 1.

Next, the partial relative growth rates $\dot{x}_{ij}/x_i$ have to be combined to the relative growth rate $\dot{x}_i/x_i$ for all j. In order to prevent the addition of fuzzy sets this combination shall be carried out in another way.
The partial relative growth rate $[\dot{x}_{ij}/x_i](k)$ is assumed to be known. Using the Euler approximation (2) the single effect $x_{ij}(k+1)$ is caused by $x_j(k)$.
The pertinent fuzzy sets of $x_i(k)$, $[\dot{x}_{ij}/x_i](k)$, and $x_{ij}(k+1)$ are composed by the fuzzy algorithm RATE (see Table 2). (Note that the classes A and H are extented to A0,A1,...,A4 and H0,H1,...,H4, respectively, to prevent saturation effects).

Instead of an addition, $x_i(k+1)$ is calculated by a sequence of single effects $x_{ij}(k+1)$. This procedure is carried out according to the following calculation scheme:

step 0: choose row i; put j=0

step 1: put $x_{io}(k+1)=x_i(k)$

step 2: j=j+1

step 3: $(x_j(k),G_{ij}) \xrightarrow{GX} [\dot{x}_{ij}/x_i](k)$

step 4: $([\dot{x}_{ij}/x_i](k),x_{ij-1}(k+1)) \xrightarrow{RATE} x_{ij}(k+1)$

step 5: if $j<n+1$ go to step 2 else

step 6: put $x_i(k+1)=x_{ij}(k+1)$

step 7: unless all rows have been chosen go to step 0.

It should be noted that the used fuzzy algorithms GX and RATE always lead to fuzzy values like A,B,..,H of the variables x_i.

In the following five examples of Volterra systems treated in the described manner are presented.
The fuzzy simulations are carried out by means of our fuzzy simulation system FUZSIM [1].
In all examples x_5 has a constant value. Furthermore, only the class having the greatest membership value is depicted for a variable.

The treated original Volterra systems are characterized as follows.

Example 1 (Fig. 1): The system has a fixpoint at (1,1,1,1,1) and a stable cycle around it.

Example 2 (Fig. 2): $x_1, x_2, x_3 \longrightarrow \infty$, x_4 goes to a large constant.

Example 3 (Fig. 3): $x_1, x_2, x_3 \longrightarrow 0$, x_4 goes to a small constant.

Example 4 (Fig. 4): The system has fixpoints on a straight line which goes through (1,1,1,1,1). The trajectories go to one of these fixpoints.

Example 5 (Fig. 5): The system is a hypercycle and has a stable cycle around (1,1,1,1,1).

The fuzzy simulation results show these pertinent qualitative behaviours. Hence, the presented approach seems to be a promising means for getting answers for qualitative behaviours and robustness with respect to changes of the parameters of the sy system considered.

Reference

[1] Straube, B.: FUZSIM - A simulation program package for fuzzy systems. ZKI-Informationen 2/1983 (techn. report of the Zentralinstitut für Kybernetik und Informationsprozesse der Akademie der Wissenschaften der DDR), Berlin.

Table 1 Operational table for GX

GX	A	B	C	D	E	F	G	H
PB	CP1	CP1	CP2	CP2	CP3	CP3	CP4	CP4
PM	CP1	CP1	CP1	CP1	CP2	CP2	CP3	CP3
PS	CP1	CP1	CP1	CP1	CP1	CP1	CP2	CP2
NU	CNU	CNU	CNU	CNU	CNU	CNU	CNU	CNU
NS	CN1	CN1	CN1	CN1	CN1	CN1	CN2	CN2
NM	CN1	CN1	CN1	CN1	CN2	CN2	CN3	CN3
NB	CN1	CN1	CN2	CN2	CN3	CN3	CN4	CN4

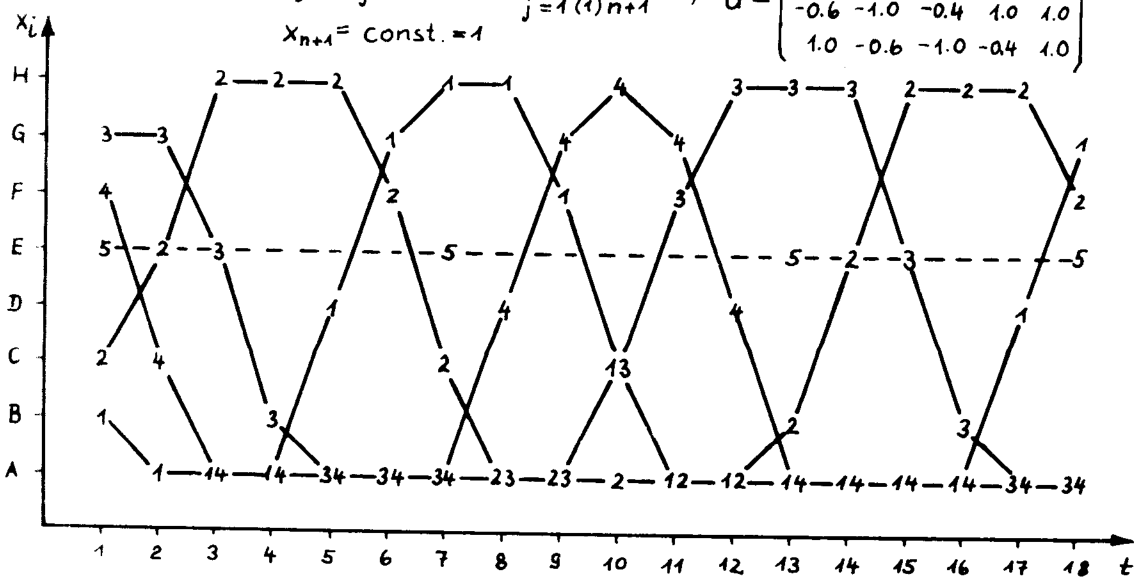
Table 2: Operational table for fuzzy algorithm RATE

RATE	A4	A3	A2	A1	A0	B	C	D	E	F	G	H0	H1	H2	H3	H4
CP4	A0	B	C	D	E	F	G	H0	H1	H2	H3	H4	H4	H4	H4	H4
CP3	A1	A0	B	C	D	E	F	G	H0	H1	H2	H3	H4	H4	H4	H4
CP2	A2	A1	A0	B	C	D	E	F	G	H0	H1	H2	H3	H4	H4	H4
CP1	A3	A2	A1	A0	B	C	D	E	F	G	H0	H1	H2	H3	H4	H4
CNU	A4	A3	A2	A1	A0	B	C	D	E	F	G	H0	H1	H2	H3	H4
CN1	A4	A4	A3	A2	A1	A0	B	C	D	E	F	G	H0	H1	H2	H3
CN2	A4	A4	A4	A3	A2	A1	A0	B	C	D	E	F	G	H0	H1	H2
CN3	A4	A4	A4	A4	A3	A2	A1	A0	B	C	D	E	F	G	H0	H1
CN4	A4	A4	A4	A4	A4	A3	A2	A1	A0	B	C	D	E	F	G	H0

Example 1:

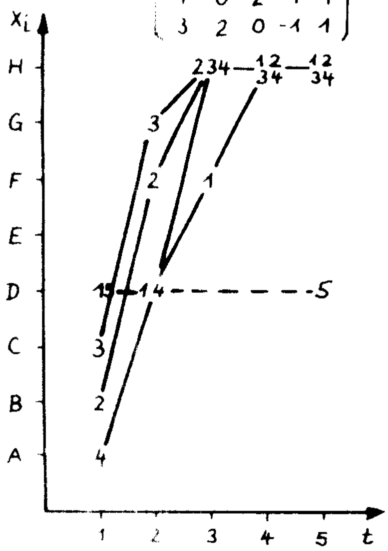
$$\frac{\dot{x}_i}{x_i} = \sum_j^n G_{ij} x_j \quad , \quad i = 1(1)n, \; j = 1(1)n+1 \quad , \quad G = \begin{pmatrix} -0.4 & 1.0 & -0.6 & -1.0 & 1.0 \\ -1.0 & -0.4 & 1.0 & -0.6 & 1.0 \\ -0.6 & -1.0 & -0.4 & 1.0 & 1.0 \\ 1.0 & -0.6 & -1.0 & -0.4 & 1.0 \end{pmatrix}$$

$$x_{n+1} = \text{const.} = 1$$



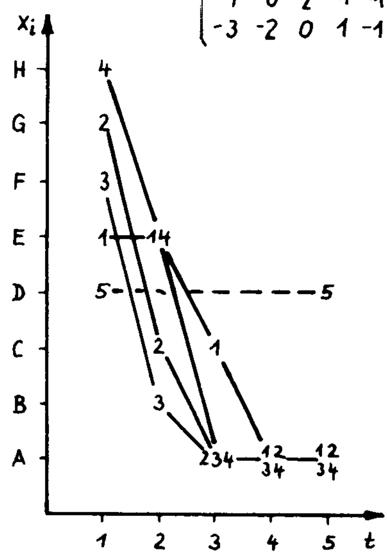
Ex.2:

$$G = \begin{pmatrix} -1 & -1 & 2 & 1 & 0 \\ 2 & 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 3 & 2 & 0 & -1 & 1 \end{pmatrix}$$



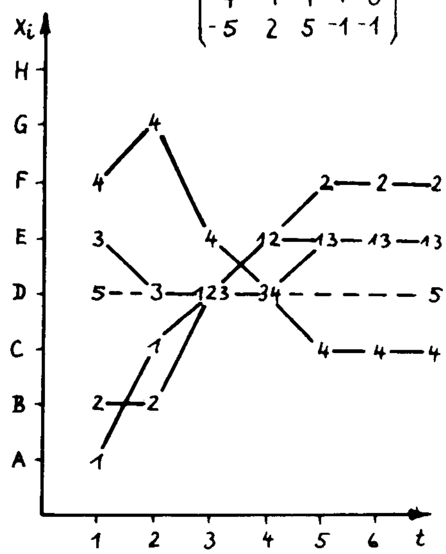
Ex.3:

$$G = \begin{pmatrix} 1 & 1 & -2 & -1 & 0 \\ -2 & -1 & -1 & 0 & -1 \\ -1 & 0 & -2 & -1 & -1 \\ -3 & -2 & 0 & 1 & -1 \end{pmatrix}$$



Ex.4:

$$G = \begin{pmatrix} -3 & 1 & 1 & 0 & 1 \\ 1 & 0 & -3 & 1 & 1 \\ 4 & -1 & -4 & 1 & 0 \\ -5 & 2 & 5 & -1 & -1 \end{pmatrix}$$



Example 5:

$$G = \begin{pmatrix} -K & K & 0 & 0 & 0 \\ -K & 0 & K & 0 & 0 \\ -K & 0 & 0 & K & 0 \\ -K & 0 & 0 & 0 & K \end{pmatrix}$$