EVALUATING FUZZY CONTROLLERS THROUGH POSSIBILITY AND CERTAINTY MEASURES

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The concept of fuzzy controller is of great importance as a tool for processing fuzzy information with fuzzy or nonfuzzy schemes of reasoning. In control engineering the fuzzy controllers form a topic of extensive studies. The papers devoted to this topic mainly are case studies presenting applications and applicational ideas for various fields of automatic control, viz. to systems controlled at present manually by a human operator. Yet, there is a certain lack of theoretical investigations concerning the background of the methods of construction and execution of the control algorithms. Here, we will add a few remarks to this theoretical side.

Usually the construction of a fuzzy controller R is starting from a set of control rules connecting inputs and outputs for some special cases of control actions. These control rules are thought of as given by suitable fuzzy subsets of corresponding input and output spaces, and as represented by linguistic labels of some linguistic variables. For input fuzzy set X and output fuzzy set U we write the control rule as usualy as: $X \Longrightarrow U$.

Let us suppose that the controller R we discuss in the following is given by the finite set

$$X_k \Longrightarrow U_k \,, \quad 1 \leq k \leq M \tag{1}$$

of control rules. For every input fuzzy set X, the output fuzzy set U of the fuzzy controller R is given by the compositional rule of inference of Zadeh (1973) as

$$U = X \circ R = R''X \tag{2}$$

where "$\circ$" denotes sup-min composition in the sense of fuzzy relational composition and "R"X" is the same composition but viewed as the fuzzified full picture of X by the fuzzy mapping R as in e.g. Gottwald (1983).

Up to now, there are essentially two ways to construct the fuzzy controller R out of the system (1) of control rules. The first one used from the very beginning by Mamdani (1974) takes

$$R = \bigcup_{k=1}^{M} (X_k \times U_k) \tag{3}$$

with the fuzzy cartesian product $X_k \times U_k$ defined by the membership values $(X_k \times U_k)(x,u) = \min(X_k(x), U_k(u))$. The second one takes R as a solution of the finite system of fuzzy relation equations

$$U_k = X_k \circ R \,, \qquad 1 \leq k \leq M \,. \tag{4}$$

Both methods have their advantages and disadvantages. Thus, e.g., it is quite simple to construct R by (3), but then it may happen that for some $1 \leq k \leq M$: $U_k \neq X_k \circ R$, i.e. the rules (1) may interact (cf. Czogala/Pedrycz (1981), Gottwald (1983)). On the other side, if R is constructed as a solution of system (4) no interaction of the rules will appear, but the solvability of (4) is not obvious - cf. e.g. Sanchez (1984), Gottwald (1984) for solvability criteria - and in case of solvability in general there exist many solutions (cf. Czogala/Drewniak/Pedrycz (1982)).

In every case, however, the system (1) of control rules is supposed to be given. And it not only may happen that those rules interact in the sense just mentioned, but some rules may "contradict" one another in the sense that some input is activating some different rules with "very different" output fuzzy sets, i.e. with (almost) contradictory control advises.

Here we will have a bit a closer look at that last named possibility. And, of course, we are interested in some means to avoid or detect such conflicting rules, or to reduce in some sense the "degree" of inconsistency of the set (1) of control rules. And we are also interested in means of evaluation and comparison of fuzzy controllers with regard to these aspects.

As an essential tool we consider the measures of possibility (cf. Zadeh (1978)) and certainty defined for fuzzy sets A,B by

$$\mathrm{Poss}(A/B) =_{\mathrm{def}} \mathrm{hgt}(A \cap B) = \sup_x \min(A(x), B(x)), \tag{5}$$

$$\mathrm{Cert}(A/B) =_{\mathrm{def}} 1 - \mathrm{Poss}(\overline{A}/B) \tag{6}$$

with $\overline{A}$ the complement of fuzzy set A. Using the logico-settheoretical notation of e.g. Gottwald (1983) we get

$$\mathrm{Poss}(A/B) = [\![\bigvee_x (x \in A \wedge x \in B)]\!] \,, \tag{7}$$

$$\mathrm{Cert}(A/B) = [\![\bigwedge_x (x \in A \longrightarrow x \in B)]\!] \tag{8}$$

with the implication operator $\longrightarrow$ in (8) characterized by: $s \longrightarrow t =$

max(1-s,t). Therefore, the possibility Poss(A/B) of B with respect to A is a degree of overlapping of A and B, and the certainty Cert(A/B) of B with respect to A is a degree of containment of A in B.

Now, suppose that the fuzzy controller is designed according to (3) from the system (1) of control rules. For any input fuzzy set X put

$$p_k(X) = \mathrm{Poss}(X/X_k) \ . \tag{9}$$

Then for the output fuzzy set $U = X \circ R = R"X$ we get for each point u of the output space

$$U(u) = (R"X)(u) = \max_{1 \leq k \leq M} \left(p_k(X) \wedge U_k(u) \right) , \tag{10}$$

thus expressing the output U in terms of the possibilities (9) to which input X is the special input fuzzy set X_k, i.e. to which input X "activates" controller rule with index k, and in terms of the outputs U_k of the control rules from (1).

To get more information on the mutual influences of the control rules (1) we may consider for all $1 \leq k,l \leq M$ the indices

$$p_{kl} = \mathrm{Poss}(X_k/X_l) \ , \qquad c_{kl} = \mathrm{Cert}(X_k/X_l) \tag{11}$$

which indicate the possibilities and certainties to which the input fuzzy set X_k of controller rule k activates rule l of (1).

Both of the matrices (p_{kl}) and (c_{kl}) are providing means for evaluating the interactions in the rules (1), e.g. also combined with some threshold level. With such a threshold level α we may derive from (11) e.g. the indices

$$p'_{kl} = \begin{cases} 1, & \text{if } p_{kl} > \alpha \\ 0, & \text{if } p_{kl} \leq \alpha \end{cases} \tag{12}$$

and use the matrix (p'_{kl}) to evaluate the rules of (1) or to change the system (1) e.g. in such a way as to eliminate all those rules whose corresponding lines (or rows because of symmetry) of this matrix contain "too much" 1's, e.g. have a sum greater than some (second) threshold level.

If, otherwise, the fuzzy controller R constructed - in some way - from rules (1) is not or only approximately a solution of system (4) of fuzzy relation equations, then for some distance function ϱ of fuzzy sets the sum

$$Q = \sum_{k=1}^{M} \varrho \, (R"X_k \, , U_k) \tag{13}$$

indicates the quality of the fuzzy controller R. Higher values of Q show also an inappropriateness of the controller designed - or of the system (1) of control rules.

The dichotomy last mentioned makes it desirable to have some tool for directly evaluating the system (1) of control rules. Here, another kind of index may be useful. To describe the principle in a simple way, put

$$m^+(t) = \begin{cases} t, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases} \qquad (14)$$

and consider as an index of conflict of the k-th and l-th control rule the value

$$d_{kl} = m^+ (\mathrm{Poss}(X_k/X_l) - \mathrm{Poss}(U_k/U_l)) \qquad (15)$$

and as a global index of conflict of controller rule k with the remaining ones

$$d_k = \sum_{l=1}^{M} d_{kl} \ . \qquad (16)$$

Again, the inconsistency of system (1) may be reduced by eliminating the control rule(s) with highest index of conflict (16).

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