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FUZZY IMBEDDING THEORY AND ITS APPLICATION*

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ABSTRACT

This paper deals with the imbedding problem in the lattices with a topology. To be more precise, it touches upon the imbedding problem in L-fuzzy topological space, where L is a fuzzy lattice. Some fundamental results such as the fuzzy unit interval, Q-neighborhood structure and algebraic properties of union-preserving maps in lattices are collected. A pointwise characterization of fuzzy complete regularity is yielded by means of the Q-neighborhood structures and some algebraic properties of certain class of maps in lattices. The Weil theorem on fuzzy uniformity and the general imbedding theorem in the fuzzy basic cube are established. As applications of the imbedding theorem, a fuzzy version of the well-known Urysohn metrizable theorem and the general theory of the fuzzy Stone-Čech compactification are given.

KEYWORDS

L-Fuzzy topology; Q-neighborhood; union-preserving map; Fuzzy uniformity; Fuzzy imbedding theorem; Fuzzy metric space; Fuzzy Stone-Čech compactification; Lattice with a topology.

INTRODUCTION

According to Ehresmann's insight[1], a lattice with the right distributivity property deserves to be studied as a generalized topological space in its own right. At his suggestion, some interesting results in this field have been obtained and their application to equivarant topology and homotopy theory have been done. A survey by Johnstone[2] well described the situation. Just as shown in that survey, the pointless approach to this field is moving toward the so-called "pointed approach". Although the works on fuzzy topology are not mentioned in the survey, the research of fuzzy topology is exactly a field to investigate the topological structure of a rather general type of lattice. Moreover, in the fuzzy topology the transition from pointless one to pointed one has been completed successfully and has been turned it to a combination of both approaches. Some deeper result, such as the imbedding theorem, has been obtained. So on the one hand, the fuzzy topology is a fundamental part of fuzzy mathematics; on the other hand, in the traditional mathematics, it is also a field which seems to be worth our attention. We hope that this paper reflects this development in fuzzy topology via the establishment of the fuzzy imbedding theory.
Let L be a completely distributive lattice with an order reversed involution. A map from an ordinary set X to L is said to be an L-fuzzy set in X. The concept of fuzzy set, taking the ordinary set as a special case, provides a foundation for treating mathematically the fuzzy phenomena which exist prevasively in our real world and for

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building new branches of fuzzy mathematics. A topological structure have been introduced in the collection of all the L-fuzzy sets in X since 1968. Fuzzy topology, which is generally considered to be a generalization of ordinary general topology, is an animated area of fuzzy mathematics. In the research of fuzzy topology, besides many simple translations from general topology to this more general setting, some significant advances have been made[3]. The most spectacular results appeared in the literature may be as follows: (1) the profound investigation on fuzzy uniformity and fuzzy metric dealt with by the so-called pointless approach[4,5] and (2) introducing the concept of fuzzy points and its neighborhood structures and a lot of "pointed approach" works arising therefrom[6-13]. Now, we try to combine these two aspects for building the important imbedding theorem. Some interesting applications of the imbedding theorem are also shown. For building the imbedding theorem we must first choose an enough nice space as standard space and look for some rather simple conditions under which a space is homeomorphic to a subspace of the standard space. Thus these simple conditions, which are usually concerned with the separation properties (e.g. completely regularity), imply a lot of nice properties shared by the standard space. This procedure is often applied to research works and usually yieldes much results. Now in the area of fuzzy topology, a nice enough space, namely fuzzy unit interval I(L), have been given by Hutton[14,15]. In this paper we shall concisely describe this space I(L). In order to investigate the desired separation properties (fuzzy completely regularity and fuzzy sub-T_0), we need to make some preparations. Through an analysis on fuzzy membership relation in fuzzy set theory we shall introduce a new kind of neighborhood structure, i.e. Q-neighborhood which is different from the traditional neighborhood in topological space[16,17]. Some algebraic operations and their properties on the class of union-preserving mappings in lattice are also explained Thus after discussing the completely fuzzy regularity and sub-T_0 spaces, we shall state an n.a.s.c., in which an L-fuzzy topological space can be imbedded in the fuzzy basic cube C(L) (i.e. the product of some fuzzy unit intervals). Along with the establishment of the fuzzy imbedding theorem, we shall apply this theorem to get a fuzzy version of well-known Urysohn metrization theorem and to expound a gereral theory of the fuzzy Stone-Čech compactification.

1. Preliminares

In this paper, X and Y denote non-empty (ordinary) sets. I denotes unit interval $[0,1]$. $(L, \leq, \wedge, \vee, ')$ denotes a fuzzy lattice, i.e. a completely distributive lattice with an order inversed involution. Its greatest element and the smallest element are denoted by 1 and 0 respectively. A map A from X to L is said to be an L-fuzzy set (or, simply, fuzzy set) in X. The collection of all the fuzzy sets in X, denoted by L^X, can be naturally seen as a fuzzy lattice $(L^X, \subseteq, \cap, \cup, ')$. Its lattice operations and involution are pointwise induced by the corresponding operations in the lattice L. The set $\{x \in X: A(x) > 0\}$ is called the support of A and is denoted by supp A. A L-fuzzy set is called a fuzzy point iff it takes the value 0 for all $y \in X$ except one, say $x \in X$. If its value at x is λ, we denote it by x_λ and λ is called its membership grade. A fuzzy topology for X is a subfamily $\mathfrak{T}$ of L^X which is closed under arbitary supremums and finite infimums. The pair $(X, \mathfrak{T})$ always denotes a fuzzy topological space with topology $\mathfrak{T}$. In this paper, if there is nothing to confuse, the adjective "fuzzy" is usually omitted in the corresponding fuzzy structures.

2. Fuzzy Unit Interval I(L)[14,15]

Consider the set $\tilde{I}$ of all monotonic decreasing map $\lambda : R \to L$ for which $\lambda(t)=1$ for $t<0$ and $\lambda(t)=0$ for $t>1$. For $\lambda, \mu \in \tilde{I}$, λ and μ is equivalent iff $\forall t \in \mathbb{R}$ $\lambda(t-) = \mu(t-)$ and $\lambda(t+) = \mu(t+)$ (where $\lambda(t+) = \bigvee_{s>t} \lambda(s)$ and $\lambda(t-) = \bigwedge_{s<t} \lambda(s)$). Then the fuzzy unit interval I(L) is the corresponding quotient set of I. The fuzzy topology $\mathfrak{T}$ on I(L) is always defined by a subbase $\{L_t, R_t : t \in \mathbb{R}\}$ where $L_t(\lambda)=$

$\lambda(t-)'$ and [illegible]$(\lambda) = \lambda(t+)$.

Under certain lattice conditions the fuzzy topology of I(L) is like the topology of the ordinary unit interval. In fact, we have

<u>Theorem 1.</u> If L is orthocomplementary, i.e. for any $a \in L$, $a \wedge a'=0$ and $a \vee a'=1$, then there exists a natural 1 — 1 correspondence $\wp$ between the open sets in the usual topology for $[0,1]$ and the open sets in the fuzzy topology for I(L) which preserves arbitrary union and finite intersections and can be defined as follows: for the open interval $(a,b) \subseteq I$, $\wp((a,b))=R_a \wedge L_b$.

Though there exists a good correspondence $\wp$ between the two families of open sets, the space I(L) have some special and important topological properties which the real unit interval does not possess. See [8,15,23]. For instance, we indicate the following

<u>Theorem 2.</u> If L is orthocomplementary, then for $\alpha \in L$, $\alpha > 0$, I(L) may not be a α^*-compact space.

For the case $\alpha = 1$, the proof of Theorem 2 is given in [15]. This proof with a slight modification is also suitable for the case $0 < \alpha < 1$.

<u>Fuzzy Membership Relation and Q-neighborhood</u>[6,9,10,17]

The symbol $\triangleleft$ denotes a fuzzy membership relation between the fuzzy points and the fuzzy sets. The notation "$x_\lambda \triangleleft A$" means that there exists relation $\triangleleft$ between the point x_λ and the set A. The negation of the relation $\triangleleft$ is denoted by $\ntriangleleft$. For example, take "belonging to" relation $\in$ as $\triangleleft$ where $x_\lambda \triangleleft A$ iff $\lambda \leq A(x)$. For this relation $\in$, the following Multiple-choice principle is not valid.

<u>Multiple-choice principle</u> Suppose that $\{A_i\}$ is a family of L-fuzzy sets. If $x_\lambda \triangleleft \vee\{A_i\}$ (the union of these A_i), then there exists a A_i such that $x_\lambda \triangleleft A_i$.

In fact, take $L=[0,1]$, $\lambda=1$, $x \in X$ and $A_n = x_{1-\frac{1}{n}}$ $(n=2,3,\ldots)$. Obviously $x_\lambda \in \vee\{A_n\}$, but $x_\lambda \notin$ any A_n. That is to say, the multiple-choice principle does not hold for the relation $\in$.

In ordinary set theory, the Multiple-choice principle is an evident and very useful fact for the ordinary "belonging to" relation, Hence the failure of this principle for relation $\in$ in fuzzy set theory may be an important cause leading to the serious limitation of corresponding neighborhood structure — the traditional neighborhood system — in the research of fuzzy topological spaces.

Now we shall establish the following four principles, which seem intuitively to be evident, to determine the reasonable fuzzy membership relation .

<u>I. Extension principle.</u> Restricting the relation $\triangleleft$ to the ordinary set theory, $\triangleleft$ will become the usual belonging relation $\in$. Precisely, for any fuzzy lattice L, if p and A is an ordinary point and an ordinary subset in X respectively, then $p \triangleleft A$ iff $p \in A$.

Since the fuzzy sets (points) take the ordinary sets (points, resp.) as special case, it is natural to put forward the Extension reinciple.

For a fuzzy point $x_\lambda = p$ and a fuzzy set A, the relation $p \triangleleft A$ or not must be determined by the relation between $p(x)=\lambda$ and $A(x)=\mu$. Since there is only order relation and involution in L, the relation between λ and μ will be described by a system of formulae about λ and μ expressed in terms of the order relation and involution. (e.g. $\lambda \not\leq \mu'$ and so on). In addition, this system of formulae will be valid not only for some pair of λ and $\mu = A(x)$, but for any fuzzy lattice L and any $\lambda \in L(\lambda \neq 0)$ and any $A(x)$ as well. According to the consideration above, we raise the following

<u>II. Value set determination principle.</u>

The fact that $x_\lambda \triangleleft A$ or not is completely determined by a system of formulae about λ and $A(x)$ expressed in terms of the order relation and involution. Moreover, the system of formulae is valid not only for some pair λ and $A(x)$, but for any fuzzy lattice and any $\lambda \in L(\lambda \neq 0)$ and any $A(x)$ as well.

[illegible] minimum principle.

[illegible] point p, p $\lhd$ X and p $\not\lhd$ $\emptyset$, where X and $\emptyset$ denote the greatest L-fuzzy [illegible] L-fuzzy set on X respectively.

IV. [illegible] choice principle.

[illegible] principle [illegible] just stated above.

[illegible] can be deduced from the principle II and III. But the principle II, [illegible] independent to each other.

[illegible] fuzzy point x_λ is said to be quasi-coincident (simple, Q-coincident) with [illegible] $\not\leq$ A'(x).

[illegible] quasi-coincident relation (simply, Q-relation) is unique fuzzy membership [illegible] satisfying the above four principles.

[illegible] fuzzy set A is said to be a quasi-coincident neighborhood (simple, [illegible]) of a fuzzy point x_λ in $(X, \mathcal{T})$ iff there an open set $U \in \mathcal{T}$ such that [illegible] x_λ is quasi-coincident with U.

[illegible] relation and the corresponding Q-neighborhood structure together [illegible] role in the research of many topics of fuzzy topology, such as convergence [illegible], product and quatient space[7], compactness[8,11] uniformity and im- [illegible] fuzzy metrizable problem[13,21] and fuzzy function space[12].

[illegible] Operations on Union-preserving Mappings in [illegible] and Fuzzy Uniformity[4,17-19]

[illegible] a uniformity for a set X is a non-void faimily of subsets of [illegible] requirements. Obviously each member D of the uniformity may [illegible] $2^X \to 2^X$ by $D(U) = \{y : x \in U \text{ and } (x,y) \in D\}$. It is apparent [illegible] for U and U_α in 2^X. Conversely, given $D: 2^X \to 2^X$ [illegible] $\bigcup D(U_\alpha)$ for U and U_α in 2^X, we may define $\tilde{D} \subseteq$ [illegible] the diagonal by $\tilde{D} = \{(x,y) : y \in D(x)\}$. Thus in defining a [illegible] basic elements of the fuzzy uniformity to be [illegible] $L^X \to L^X$ which satisfy: (1) D is increasing, i.e. for [illegible] (2) D is unionpreserving, i.e. for $\lambda_i \in L^X$, $D(\bigvee \lambda_i)$ = [illegible]

[illegible] by a fuzzy uniformity we need to study some algebraic [illegible] operation and inverse operation) on the class of union- [illegible] L^X to L^X.

[illegible] L_2 be completely distributive lattices. Map $f: L_1 \to L_2$ will [illegible] iff $a \geq b$ implies $f(a) \geq f(b)$ for any $a, b \in L_1$. Map $f: L_1 \to$ [illegible] called union-preserving iff $f(\bigvee a_j) = \bigvee f(a_j)$ for any $a_j \in L_1$ ($j \in$ [illegible]

[illegible] $\in$ L. A set $B \subseteq L$ is called the minimal set relative to a iff (1) [illegible] for every set $A \subseteq L$ satisfying sup A=a and for every $b \in B$, there [illegible] that $b \leq d$.

[illegible] pointed out that for each element $a \in L$, there exists a minimal set [illegible]

[illegible] L_1 and L_2 be complete lattices, $f: L_1 \to L_2$ an order-preserving map. [illegible] is defined by

$$f^*(a) = \bigwedge_{\sup D = a} \left[\bigvee_{d \in D} f(d) \right] \qquad \forall a \in L_1 .$$

[illegible] L_2 [illegible] the completely distributive law, then

[illegible] (b), where B is minimal set relative to a.

[illegible] for every $a \in L_1$, $f^*(a) \leq f(a)$.

[illegible] is a union-preserving map.

(4) f^* is the greatest $g: L_1 \to L_2$ which takes values less than or equal to that of f and is union-preserving.

Definition [illegible] Let L_1 and L_2 be the complete lattices, $f_1, f_2: L_1 \to L_2$ be union-preserving maps. If L_1 satisfies completely distributive law, we define $f_1 \cap f_2$, $f_1 \wedge f_2$: $L_1 \to L_2$ by

$$(f_1 \cap f_2)(a) = f_1(a) \wedge f_2(a) \qquad a \in L_1 ,$$
$$f_1 \wedge f_2 = (f_1 \cap f_2)^*$$

The map $f_1 \wedge f_2: L_1 \to L_2$ is called the intersection of f_1 and f_2.

Since $f_1 \cap f_2$ is obviously order-preserving, $(f_1 \cap f_2)^*$ is well-defined. Thus by Proposition [illegible] the intersection $f_1 \wedge f_2$ is still union-preserving.

Theorem [illegible] Let L_1 and L_2 be completely distributive lattices, $f_1, f_2: L_1 \to L_2$ be union-preserving maps. Then for each $a \in L$,

$$(f_1 \wedge f_2)(a) = f_1(a) \wedge f_2(a) \bigwedge \Big(\bigwedge_{a_1 \vee a_2 = a} \left[f_1(a_1) \vee f_2(a_2) \right] \Big).$$

Furthermore, the above formula can be simplified as

$$(f_1 \wedge f_2)(a) = \bigwedge_{a_1 \vee a_2 = a} \left[f_1(a_1) \vee f_2(a_2) \right]$$

iff $f_1(0)$ [illegible].

The collection of all the union-preserving maps from L_1 to L_2 will denote $\mathcal{Q}(L_1, L_2)$. When [illegible], we write simply $\mathcal{Q}(L_1, L_2)$ as $\mathcal{Q}(L)$.

Definition [illegible] Let $f: L_1 \to L_2$ be union-preserving. Define its inverse $f^{-1}: L_2 \to L_1$ by

$$f^{-1}(a) = \inf \{ b \in L_1 : f(b') \leq a' \} \qquad a \quad L_2 .$$

Theorem [illegible] Let $f: L_1 \to L_2$ and $g: L_2 \to L_1$ be union-preserving maps Then $g = f^{-1}$ iff the following conditions hold:

(1) When [illegible], $g(b') \leq a' \longleftrightarrow f(a) \leq b$;

(2) When [illegible], $g(b') = a' = 1 \longleftrightarrow$ or $f(0) \not\leq b$ either $f(d) \leq b$ holds only for $d=0$.

Furthermore, [illegible] $f(0)=0$, the above result can be simplified as follows: $g=f^{-1}$ iff the following condition holds: $g(b') \leq a' \longleftrightarrow f(a) \leq b$.

Proposition 2. Let L_1, L_2 and L_3 be fuzzy lattice, $f, g: L_1 \to L_2$ and $g: L_2 \to L_3$ be union-preserving maps. Then

(1) f^{-1}: [illegible] is union-preserving map and $f^{-1}(0)=0$.

(2) [illegible] and if $f(0)=0$, then $(f^{-1})^{-1} = f$.

(3) if $f \leq g$, then $f^{-1} \leq g^{-1}$. Conversely if $f^{-1} \leq g^{-1}$ and $f(0)=g(0)=0$, then $f \leq g$.

(4) $(h \circ f)^{-1}$ [illegible] $f^{-1} \circ h^{-1}$ and if $h(0)=0$, then $(h \circ f)^{-1} = f^{-1} \circ h^{-1}$

(5) $(f \wedge g)^{-1}$ [illegible] $f^{-1} \wedge g^{-1}$.

Note [illegible] The [illegible] of Theorem 5 and Proposition 2 above are more general than the original ones given in [20] . The corresponding results in [20] hold only for the case when the union-preserving maps are normal, i.e. they maps 0 to 0.

Let the [illegible] $G: L_1 \to L_2$ and $H: L_2 \to L_1$ be union-preserving. We can define a corresponding $\Omega: \mathcal{Q}(L_2) \to \mathcal{Q}(L_1)$ as follows: For each $g \in \mathcal{Q}(L_2)$, $\Omega(g)(a) = HgG(a)$, $a \in L_1$. We have

Proposition [illegible]

(1) $\Omega(g) \in \mathcal{Q}(L_1)$,

(2) [illegible] $g_i \in \mathcal{Q}(L_2)$ $(i=1,2)$,

$$\Omega(g_1 \wedge g_2) \leq \Omega(g_1) \wedge \Omega(g_2).$$

[illegible] is minimal-set-preserving and $H: L_2 \to L_1$ is finite intersection-[illegible] $q_i \in \mathcal{Q}(L_2)$ $(i=1,2)$,

$$\Omega(q_1 \wedge q_2) = \Omega(q_1) \wedge \Omega(q_2) .$$

[illegible] maps Ω and H satisfy the following conditions:

[illegible] $\forall\ b \in L_2$,

[illegible] $\forall\ a \in L_1$,

[illegible] $\forall\ b \in L_2$,

[illegible] $\mathcal{Q}(L_2)$,

Ω [illegible] $(\Omega \cdot H)^{-1}$.

[illegible] the notion of fuzzy uniformity for X with the aid of these [illegible] collection of all increasing and union-preserving maps in L^X is [illegible] $\mathcal{H}(X)$.

[illegible] fuzzy quasi-uniformity on a set X is a subset $\mathcal{D}$ of $\mathcal{H}(X)$ such [illegible]

[illegible] $D \le E \in \mathcal{H}(X)$ implies $E \in \mathcal{D}$.

[illegible] $E \in \mathcal{D}$ implies $D \wedge E \in \mathcal{D}$.

[illegible] there exists $E \in \mathcal{D}$ such that $E \circ E \le D$.

[illegible] uniformity $\mathcal{D}$ is a fuzzy uniformity if it also satisfies

[illegible] implies $D^{-1} \in \mathcal{D}$.

[illegible] $\mathcal{D}$ generates a fuzzy topology $\mathcal{T}_{\mathcal{D}}$ as follows:

[illegible] there exists $D \in \mathcal{D}$ such that $D(V) \subseteq U \}$, $U \in L^X$.

[illegible] $(X, \mathcal{T})$ is said to be fuzzy uniformizable iff there is [illegible] X such that $\mathcal{T} = \mathcal{T}_{\mathcal{D}}$.

[illegible] Completely Regularity and Fuzzy Sub-T_0 Spaces [4,9]

[illegible] is said to be a completely fuzzy regular space iff for each [illegible] family $\{W_\alpha\}$ such that $U = \bigcup \{W_\alpha\}$ and for each W_α, [illegible] $f_\alpha : (X, \mathcal{T}) \to I(L)$ such that

$$W_\alpha \subseteq f_\alpha^{-1}(L_1') \subseteq f_\alpha^{-1}(R_0) \subseteq U.$$

[illegible] fuzzy uniformizable iff it is completely fuzzy regular.

[illegible] of the famous Weil theorem. It was first given in [4], but [illegible] in the original proof. A sound proof was given in [9] . Now [illegible] fuzzy completely regularity with the aid of some deeper results on [illegible] first, about the decomposition $\{W_\alpha\}$ of U in Definition 8 we [illegible] W_α is open fuzzy set, namely we have

[illegible] $(X, \mathcal{T})$ is a completely fuzzy regular space iff for each $U \in \mathcal{T}$, [illegible] open fuzzy sets $\{W_\alpha\}$ such that $U = \bigcup \{W_\alpha\}$ and continuous [illegible] $\to I(L)$ satisfying

$$W_\alpha \subseteq f_\alpha^{-1}(L_1') \subseteq f_\alpha^{-1}(R_0) \subseteq U .$$

[illegible], each open fuzzy set U is required to decomposite into a family of [illegible] these decompositions have some uncertainty which is inconvenient in application. [illegible] particular, to find the imbedding map from a space to the standard [illegible] give a correspondence between their points. In doing this we are [illegible] the concept of fuzzy point. By means of the Q-neighborhood structure and [illegible] algebraic properties of union-preserving maps in lattices we get the following [illegible] wise characterization of fuzzy complete regularity.

Theorem [illegible] $(X, \mathcal{T})$ is fuzzy completely regular space iff for each fuzzy point e and its Q-neighborhood B, there exists a Q-neighborhood A of e and a continuous map f: $(X, \mathcal{T}) \to I(L)$ such that $A \subseteq f^{-1}(L_1') \subseteq f^{-1}(R_0) \subseteq B$.

[illegible] pointwise characterization we can show that the property of fuzzy [illegible] is productive. Precisely we have the following theorem. About [illegible] product of fuzzy topological spaces and corresponding some elementary [illegible] can refer to [7].

Theorem [illegible] Suppose that for each $\alpha \in J$, $(X_\alpha, \mathcal{T}_\alpha)$ is a fuzzy completely regular [illegible] product space is also a fuzzy completely regular space.

[illegible] general topology, to establish the imbedding theorem, besides the completely [illegible] we have to add other separation requirement such as T_1 separation [illegible] the fuzzy set theory we introduced a new kind of separation as follows.

Definition [illegible] $(X, \mathcal{T})$ is said to be fuzzy sub-T_0 space iff for any pair of distinct [illegible] there exists $\lambda \in L$ $(\lambda \neq 0)$ such that for the fuzzy points x_λ and y_λ [illegible] $\notin \overline{y_\lambda}$ or $y_\lambda \notin \overline{x_\lambda}$.

[illegible] I(L) is a sub-T_0 space, but it is neither quasi-T_0 nor fuzzy [illegible]. Any subspace of a sub-T_0 space is sub-T_0.

Proposition [illegible] be an index set. Suppose that for each $\alpha \in J$, $(X_\alpha, \mathcal{T}_\alpha)$ [illegible] their product space. Then:

[illegible] $\mathcal{T}_\alpha$) is sub-T_0, then the product space $(X, \mathcal{T})$ is sub-T_0.

[illegible] ordered, $(X, \mathcal{T})$ is sub-T_0 iff each $(X_\alpha, \mathcal{T}_\alpha)$ is sub-T_0.

[illegible] regular space is said to be a fuzzy Tychonoff space iff it is a sub-T_0 [illegible], a subspace of a fuzzy Tychonoff space is still a Tychonoff space. [illegible] Proposition 5, it follows directly that the product space of some [illegible] is still a fuzzy Tychonoff space. As I(L) is a fuzzy uniform [illegible], hence both I(L) and the fuzzy basic cube C(L) (the product [illegible]) are fuzzy Tychonoff space.

[illegible] Imbedding theorem [9]

[illegible] — the fuzzy basic cube C(L) has been described and the [illegible] properties — fuzzy completely regularity and sub-T_0 have been [illegible] establish the imbedding theorem as follows:

[illegible] $\mathcal{F}$ be a family of continuous maps, where each member $f \in \mathcal{F}$ [illegible] $\mathcal{T}_f$). Suppose that $(Y, \mathcal{U})$ is the product space of these [illegible]

[illegible] defined by $E(x)_f = f(x)$, $x \in X$, is called the evaluation map.

[illegible] $\mathcal{F}$ [illegible] distinguishes points iff for each pair of distinct points x and y there is [illegible] $\mathcal{F}$ [illegible] that $f(x) \neq f(y)$.

[illegible] $\mathcal{F}$ [illegible] distinguishes points and closed sets iff for each closed fuzzy set A and [illegible] satisfying $e \notin A$ there is f in F such that $f(e) \notin \overline{f(A)}$.

[illegible] (Embedding lemma). Let $\mathcal{F}$ be a family of continuous maps, each member [illegible] $(X, \mathcal{T})$ to $(Y_f, \mathcal{T}_f)$. Then:

[illegible] map E is fuzzy continuous.

[illegible] $\mathcal{F}$ distinguishes points and closed sets, then E is an open map from X into [illegible] (endowed with relative fuzzy toplology).

[illegible] $\mathcal{F}$ distinguishes points, then the map E is one to one.

[illegible] theorem reduces the problem of embedding a space topologically in a [illegible] C(L) to the problem of looking out enough many maps from the space [illegible]

Proposition [illegible] Suppose that the family $\mathcal{F}$ distinguishes points and closed sets. If $(X, \mathcal{T})$ is sub-T_0, then $\mathcal{F}$ distinguishes points.

Proposition [illegible] Suppose that the map $f: (X, \mathcal{T}) \to I(L)$ is fuzzy continuous, $A \in L^X$, [illegible] $\in \mathcal{T}$ and $A \subseteq f^{-1}(R_0) \subseteq U$. If fuzzy point x_λ is Q-coincident with A, then [illegible] $\notin$ [illegible].

Proposition [illegible] Let $(X, \mathcal{T})$ be a fuzzy completely regular space. Then the family of all continuous maps from $(X, \mathcal{T})$ to I(L) distinguishes points and closed sets.

[illegible] the following as the chief result.

[illegible] $(X, \mathcal{T})$ is a fuzzy Tychonoff space iff $(X, \mathcal{T})$ can be imbedded in the [illegible].

[illegible] Fuzzy Metrization Problem [5,13,21]

[illegible] A fuzzy pseudo-metric (simply, p-metric) on X is a family of maps [illegible] $(0, \infty)$) satisfying the following requirements:

[illegible]

[illegible] preserving and increasing.

[illegible] $\forall\, r,s > 0$.

[illegible] $(\lambda) = D_r(\lambda)$.

[illegible]

[illegible] (X, D_r) is called fuzzy metric space iff it is sub-T_0 space.
[illegible] of maps $\{D_r\}$ is also a symmetric base of certain fuzzy uniformity [illegible] the topology $\mathcal{T}_{\mathcal{D}}$ as the corresponding metric topology.
[illegible] is just motivated by the well-known fact in general topo[illegible] is p-metrizable iff its uniformity has a countable base.
[illegible] subspace of a fuzzy p-metric space is p-metrizable.
[illegible] $\mathcal{T}$) is a fuzzy topological space. Consider a relation $\sim$ between [illegible]: $x \sim y$ iff $\forall \lambda \in L - 0$, $x_\lambda \in \overline{y_\lambda}$ and $y_\lambda \in \overline{x_\lambda}$.
[illegible] equivalence relation, hence there is a decomposition of X and a [illegible] quotient space $(\tilde{X}, \tilde{\mathcal{T}})$. It is easy to verify that $(\tilde{X}, \tilde{\mathcal{T}})$ is [illegible] call $(\tilde{X}, \tilde{\mathcal{T}})$ the associated sub-T_0 space of $(X, \mathcal{T})$.
[illegible] that $(X, \mathcal{T})$ is a fuzzy topological space with countable topo[illegible] (1) $(X, \mathcal{T})$ is fuzzy metrizable iff it is fuzzy completely [illegible] space. (2) If $(X, \mathcal{T})$ is fuzzy completely regular space, then [illegible] space is fuzzy metrizable.

[illegible] Stone-Čech Compactification [22]

[illegible] thoroughly investigated in the general topology. But it seems that the [illegible] notion in the area of fuzzy topology is not so effective as in [illegible]. Many kinds of fuzzy compactness have appeared in the literature. [illegible], the N-compactness introduced in [11] by Wang seems to be one of the most [illegible] compactness. The N-compactness is defined via fuzzy nets [6]. Notice [illegible] paragraph we take L=I.
[illegible] L=I. (1) Suppose that $S = \{S(n), n \in D\}$ is a fuzzy nets in $(X, \mathcal{T})$, [illegible] set and S(n) is fuzzy point with the membership grade λ_n. S [illegible] net $V(S) = \{\lambda_n, n \in D\}$ in the half open interval $(0\ 1]$. If V(S) [illegible] real number $\alpha \in I - \{0\}$ with respect to usual topology of $(0,1]$, [illegible] S is an α-net. (2) Fuzzy set A is called N-compact iff each α-net [illegible] at least a fuzzy cluster point with membership grade α in A. (3) When A=X is N-compact, we call $(X, \mathcal{T})$ N-compact fuzzy topological space.
Theorem [illegible] The product space of some N-compact spaces is N-compact.
Theorem [illegible] When L=I, the fuzzy unit interval I(L) and fuzzy basic cube C(L) are N-compact.
Many discussions on the fuzzy Stone-Čech compactification appeared in the literature [illegible] a special kind of fuzzy topological space, called topologically generated [illegible]. Now, based on the imbedding theorem and some results about N-compac[illegible]

[illegible] fuzzy topological space (Y, $\mathcal{U}$) is said to be a compactification [illegible] there exists a homeomorphism f from (X, $\mathcal{T}$) into (Y, $\mathcal{U}$) and there [illegible] compact fuzzy set B in (Y, $\mathcal{U}$) such that f(X)=B and supp B=Y.
[illegible] that (X, $\mathcal{T}$) is a fuzzy Tychonoff space. Then there exists a [illegible] β(X) of (X, $\mathcal{T}$) such that each fuzzy continuous map from (X, $\mathcal{T}$) [illegible] cube C(L) can be extended to β(X).

[illegible]

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