MULTI ASPECTS DECISION PROCEDURES BASED ON MINIMUM CHOICES FUZZY OPERATIONS

B. Apolloni - A. Donnarumma

SUMMARY - A selection method is presented which is based on fuzzy-sets procedures and a new procedure is proposed for reproducing membership functions.

INHALTSANGABE - Eine auf fuzzy-sets Theorie gegruendete Auswahlsmethode wird besprochen und ein Verfahren zur Darstellung der membership function wird vorgeschlagen.

SOMMARIO - Viene discusso un criterio di scelta fondato sulla teoria dei fuzzy-sets e viene proposto un procedimento per la rappresentazione della funzione di appartenenza.

Description of the proposed procedure. A procedure is proposed based on a small number of extremely simple hypoteses and constituted by very elementary operations ,in order to be shared by the largest part of the "rest of the world".
Whith reference to the classical definition of fuzzy-set /1/ we consider only stepwise membership functions(fig.2a,b)with an undetermined ordinates scale.So the job of the designer consists in rationalising the membership function into a lattice constituted by a finite (generally very small) number of ordered values, representing the hights of the bars of the diagram.
We suppose that more complex membership functions can be splitted in an analytical part,i.e. shape and parameters.
The first one - e.g. linear shape, bicubic spline and so on - is the follow-out of some theory and therefore can be taken into account in yhat framework; the second one are fuzzy parameters

and therefore they are the items of the proposed procedure. Fuzzy-sets are merged in Rn in a different way from the one generally followed in literature/2/.In this paper indeed the product of the stepwise diagrams is represented in Rn by a set of n-dimensional cells.More in details, we refer to a Rn whose coordinate axes of the previous stepwise diagrams; the product of n fuzzy-sets generates a set of cells whose edges are the lenght of the steps, one from each initial diagram, in any combination , and each cell is affectedby a level vector whose components are the membership function walues of the corresponding steps.With reference to fig.3, the product of P and **B** give rise to set of cells P.**B**.

Two problems arise to read such a picture:1) to draw a scalar walue from the levels vector;2) to put in any correspondence the scales of the coordinates axes.It must be pointed out that both the points are matter of choise for the designer, so that what follows is no more than a likely viewpoint of the authors. It could be reasonable to assume equal weights of the levels in each vector,and, at same time, equal weights of the coordinates scales, once normalized to a same maximum value.

But other criteria could be the same subjectively assessed by the designer.

Equal wheigts of the level components allow to syntetize them by their product, equal coordinate scales allow to use simple matrice in Rn.

How to compare these multidimensional fuzzy-sets in again the designer's job.Really the semantics of the fuzzy-set-theoretical operators generally involved in this step/3/assume heavvy hypoteses, such as symmetry, associativity, and so on, which are nct necessarily agreed upon by the"rest of the world".

For clearness'sake, the authors think it is more useful to approach the problem under the point of view of pattern recognition.In such a frame, the designer formulates as a natural fall-out of the statement of the fuzzy-sets, the patterns of the "yes" decision and the one of the "no" decision, so that the last job is to recognize one of the previous patterns.

Applications example. The proposed procedure has been implemented to deal with multi-aspect design problem actually arosen in selecting the bearings for a marine engine.
Suggestion exist in literature about the parameters of such bearings based essentially on the experience acquired with respect the tree following aspects: cost of materials and/or size, life, cost of manifacturing. Each aspect will be affect by a fuzzy attribute (importance attribute) meaning the importance of aspect with respect to the design problem in hand , each aspect will be evaluated by a fuzzy attribute (merit attribute) with to each aspect.
In correspondence of the available design alternatives, the following matrix alternative/aspect has been drawn (tab. 1)

	A_1	A_2	A_3	W
C_1	B	M	G	p
C_2	O	O	G	v
C_3	O	G	B	i

$A_1; A_2; A_3$ = Alternatives
C_1 = Size
C_2 = Life
C_3 = Manifacturing cost

G = Good
O = Optimal
M = Mean
B = Bad

W = Weight
p = little important
v = very important
i = important

The problem has been treated following two approaches: a classical one/4,5/ and the proposed one.
In the first case , we suppose in a realistic framework/5/that the fuzzy sets supplied by the designer are linear functions (fig.1).we start from th non-fuzzy /4/evaluation z_i of the i-th alternative

$$z_i = \frac{\sum_j x_{ij} y_j}{\sum_j y_j} = g(v_i)$$

where x_{ij} is the non-fuzzy walue meaning the merit of the i-th

aspect, y_j is the fuzzy evaluation of the importance of the j-th aspect $v_i = (x_{i1}, x_{i2}, x_{i3}; y_1, y_2, y_3)$.
The corresponding mapping from the space R_{3x3} of v_i into the space R_1 of z_i when dealing in fuzzy context , is done by operator

$$m(z_i) = \max_{g(v_i)=z_i} \cdot \left(\min \left(\min_{j=1.3} (x_{ij}), \min_{j=1.3} (y_j)\right)\right) \qquad v_i \in R_g; z_j \in R_1$$

where $m(z)$ is the membership function of z.
The resulting membership function of the tree alternative are shown in fig.1a.in fig.1b there are shown the corresponding fuzzy-set generated on the basis of the non preferability measure

$$u_i = z_i - 1/2 \sum_{\substack{i=1 \\ j \neq i}}^{3} z_j$$

From the two pictures appears a global smaller goodness of the alternative A_3 but is not easy to distinguish between A_1 and A_2.
The proposed approach requires a new statement of the initial fuzzy-sets.With respect to the ones of fig.1, the maximum points have been substituted, more realistically, with maximum segments and the ramps have been substituted by steps of height increasing with the absolute value of their gradients.Really these last parameters are the fuzzy parameters which have to be taken into account in the final choice.
Since the ordinate's scale is undeterminated, the heights of the steps ,once ordered in growing order, have been kept equal to the product of its positional index by any standard height.In R_2 each alternative is represented by tree domains which are the product of the fuzzy-sets of the importance attribute by the corresponding fuzzy-set of the merit attribute.Each domain contains a set of cells characterized by constant values of both the membership functions of the initial diagrams.it has been assumed equal weight of the normalized scales of the axes, and the membership function value of the cell equal to the product of the

two initialmembership function values.The so obtained picture of fig.3 was interpreted by the Auhtors as follows.

All the alternatives show highly weighting areas around the right upper corner of the picture,i.e. the point affected by the highest values of importance and merit.so a measure of preferibility among the alternatives was assumed to be the dispersion of the three bi-dimensional fuzzy-sets with respect to the cell with maximum mem-bership function near the right-uppermost corner.Because of the special meaning of the membership function in this case,it was assumed as dispersion measure the static moment of the membership function.

In such a way the tree alternatives are clearly distinguished, since the rate of this preferibility measure was 1, 1.3, 2.2 in correspondence respectively of A_1 , A_2 , A_3.

References

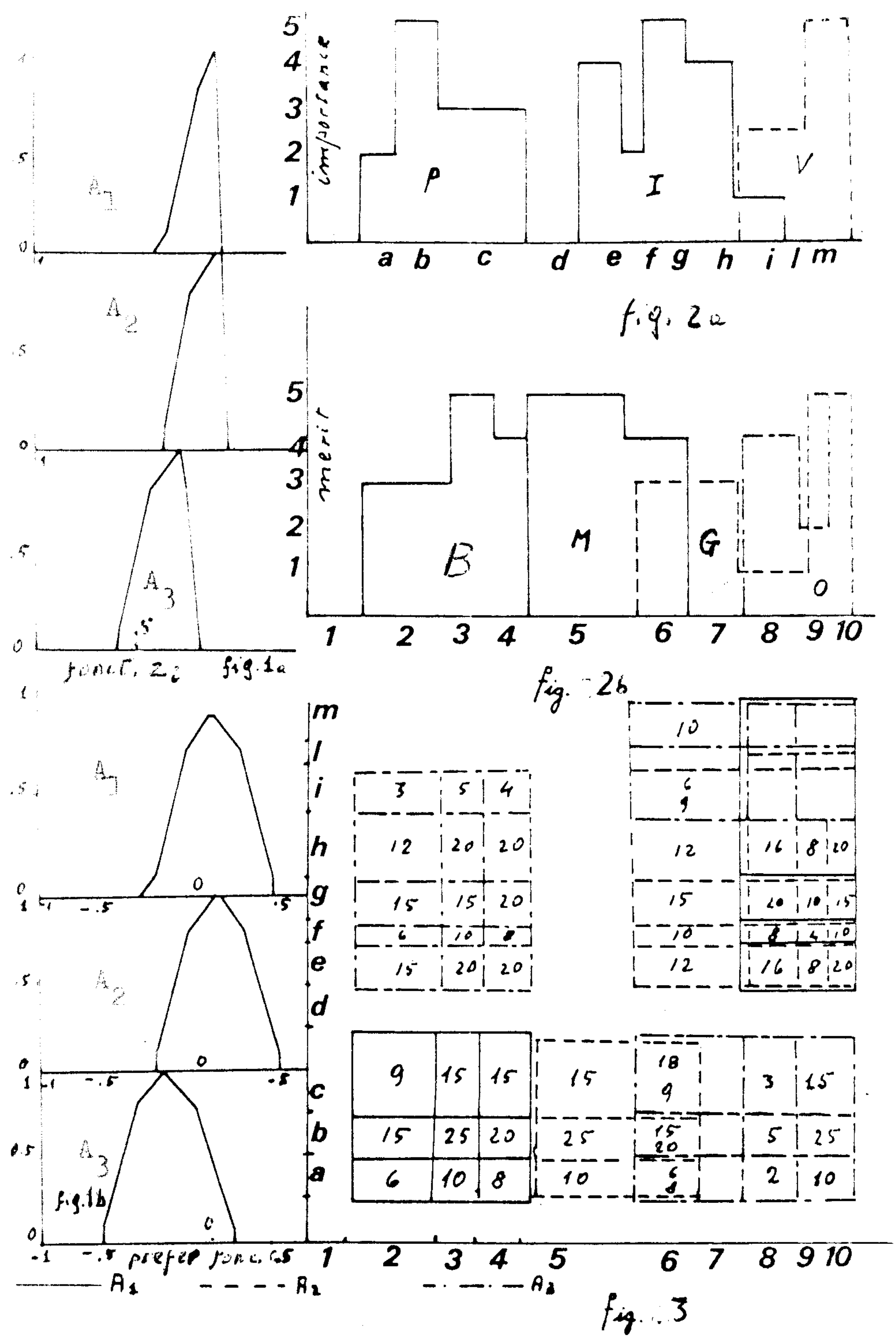
1. Zadeh,L.A., Fuzzy-sets, Inf.Contr. 8,338,353 (1965)
2. Zadeh,L.A., Calculus of Fuzzy Restrictions, in "Fuzzy sets and their applic tions to cognitive and decision processes", Zadeh et al.edt.,Academic Press (1975)
3. Dubois,D. ,Prade, H. , New results about properties and se-mantics of fuzzy-set theoretic operators, in "Fuzzy-sets, Theory and appl. to policy anal. and Inform.Systems",Wang [illegible] and S.K.Chang edt.s, Plenum Press N.Y.(1980)
4. [illegible] S.K. and Kawakernaak H.,Rating and Ranking of multiple [illegible] altern tives using fuzzy-sets,"Automatica"13,47,58(77)
5. [illegible]oni,B. and Donnarumma,A., Un approccio fuzzy alla proget-tazione meccanica ,"Progettare",Milano,n.14-15,p.65-68(1981)
6. [illegible]oni,B. and Di Gregorio,S., Arbitrary order in determini-stic algorhitms for NPc problems. Submitted at ICALP 83

Adresses of Authors

[illegible], B., Dipartimento dei Sistemi, Università della Calabria I-87036 Arcavacata di Rende (Cosenza)

[illegible], A.,Facoltà d'Ingegneria ,P.le Tecchio,I-80125 Napoli

[illegible] at ICED 83 - Kopenaghen, August 1983



importance
P
I
V
a b c d e f g h i l m
fig. 2a
merit
B
M
G
O
1 2 3 4 5 6 7 8 9 10
fig. 2b
A1
A2
A3
fonct. Zi
fig. 1a
fig. 1b
prefer fonct. S
m l i h g f e d c b a
A1
A2
A3
fig. 3