# Pansystems Clustering Analysis and Application to Fuzzy Clustering Analysis

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This paper reveals the relation between pansystems clustering analysis and fuzzy clustering analysis, shows that pansystems logic conservations and equivalence between them, then introduces many results got from pansystems clustering analysis into fuzzy clustering analysis.

Definition 1. We call $f$ is a relation function on $G$ if $f$ is a mapping from $[0,1]$ into $P(G^2)$, and call $f$ a $F$ relation function on $G$ if

1. $f$ is a relation function on $G$.
2. $f(x') \leq f(x'')$ iff $x'' \leq x'$.
3. If $(x,y) \in G^2$, then there exists a number $x'(x,y)$ such that $(x,y) \in f(x')$, $(x,y) \notin f(x'')$ if $x'' > x'(x,y)$. We denote $F(G)$ by the set of all $F$ relation functions on $G$, and $F'(G)$ by the set of all fuzzy relations on $G$.

Theorem 1. $|F(G)| = |F'(G)|$.

Proof Let $t \in F'(G)$, $f \in F(G)$,

$$\theta:\ t \longrightarrow f_t,\ f_t(x') = \{(x,y) / u_t(x,y) \geqslant x'\},$$

$$\theta':\ f \longrightarrow t_f,\ u_t(x,y) = x'(x,y).$$

It is easy to see $f_t \in F(G)$, $t_f \in F'(G)$, $\theta(\theta'(f)) = f$, $\theta'(\theta(t)) = t$. This completes the proof.

Theorem 2. If $t_i \in F'(G)$, $f_i \in F(G)$, $\theta(t_i) - f_i$, then

$$\theta(t_1 \cup t_2) = f_1 \cup f_2,\ \theta'(f_1 \cup f_2) = t_1 \cup t_2,$$

$$\theta(t_1 \cap t_2) = f_1 \cap f_2,\ \theta'(f_1 \cap f_2) = t_1 \cap t_2,$$

$$\theta(t_1 \circ t_2) = f_1 \circ f_2,\ \theta'(f_1 \circ f_2) = t_1 \circ t_2,$$

$$\theta(t_1^{-1}) = f_1^{-1},\ \theta'(f_1^{-1}) = t_1^{-1},$$

$$\theta(\bar{t}_1)(x) \supset \bar{f}_1(1-x).$$

Definition 2. We denote the classes of binary relations on $G$ satisfying reflexity, symmtry, transitivity, equivalence, semi- equivalence by $R[G]$, $S[G]$, $T[G]$, $E[G]$, $E[G]$, the classes of fuzzy relations by $R'[G]$, $S'[G]$, $T'[G]$, $E'[G]$, $E'[G]$.

$F \in A[G)$ implies $f(x) \in A[G)$ for each $x \in [0,1]$.

Theorem 3. If $t_1, t_2 \in F'(G)$, $f_i = \theta(t_i)$, $i = 1, 2$, then

1. $t_1 \leq t_2$ if and only if $f_1 \subset f_2$.
2. $t_1 \in A'[G)$ if and only if $f_1 \in A[G)$ for each $A \in \{R, S, T, E, E_s,\}$.

Definition 3. Suppose that f is a F relation function on G, we define pansystems operation $\varepsilon_i$, $\delta_i$ $(i = 1, \ldots, 5)$ as follows

$$\varepsilon_1(f) = f \vee f^{-1} \vee I, \qquad \delta_1(f) = \varepsilon_1(f)^+,$$
$$\varepsilon_2(f) = f \wedge f^{-1} \vee I, \qquad \delta_2(f) = \varepsilon_2(f)^+,$$
$$\varepsilon_3(f) = f^{-+} \wedge f^+ \vee I, \qquad \delta_3(f) = \varepsilon_3(f)^+,$$
$$\varepsilon_4(f) = f \circ f^{-1} \vee I, \qquad \delta_4(f) = \varepsilon_4(f)^+,$$
$$\varepsilon_5(f) = f^{-1} \circ f \vee I, \qquad \delta_5(f) = \varepsilon_5(f)^+.$$

Theorem 4. If $t_\sigma \in A[G)$, $A \in \{R', S', E'_s\}$, then $\vee t_\sigma \in A[G)$. If $A \in \{R', S', T', E', E'\}$, , then $\overset{+}{\vee} t_\sigma \in A[G)$.

Theorem 5. If $t \in T'[G)$, $\theta \in \{(n), -1, t\}$, then $t^\theta \in T'[G)$.

Theorem 6. $\varepsilon_i(t) \in E'_s[G)$, $\delta_i(t) \in E'[G)$, $i = 1, \ldots, 5$.

Theorem 7. $\varepsilon_i(t^{-1}) = \varepsilon_i(t)$, $\delta_i(t^{-1}) = \delta_i(t)$, $i = 1,2,3$.

Theorem 8. If $t \leq t'$, then $\varepsilon_i(t) \leq \varepsilon_i(t')$, $\delta_i(t) \leq \delta_i(t')$, $i = 1, \ldots, 5$.

Theorem 9. $\varepsilon_i(\wedge t_\sigma) \leq \wedge \varepsilon_i(t_\sigma)$, $\vee \varepsilon_i(t_\sigma) \leq \varepsilon_i(\vee t_\sigma)$ $\delta_i(\wedge t_\sigma) \leq \wedge \delta_i(t_\sigma)$, $\vee \delta_i(t_\sigma) \leq \delta_i(\vee t_\sigma)$, $i = 1, \ldots, 5$.

Theorem 10. $\delta_2(t) \leq \delta_3(t) \leq \delta_1(t)$.

Theorem 11. $\delta_1(t) = \delta_1(t \cup t^{-1})$, $\delta_2(t) = \delta_2(t \cap t^{-1})$, $\delta_3(t) = \delta_3(t^+)$.

Theorem 12. $\delta_1(t^{(n)}) \leq \delta_1^{(n)}(t)$.

Theorem 13. If $t, t' \in E'[G)$, $t \circ t' = t' \circ t$, then $t \circ t' \in E'[G)$, $t \circ t' = t \overset{+}{\vee} t'$.

Theorem 14. If $t, t' \in S'[G)$, $t \circ t' \leq t' \circ t$, then $t \circ t' = t' \circ t$

References


/1/ Wu Xuemou, Pansystems Methodology: Concepts, Theorems and Applications (V- VI), Science Exploration, 4(1983),1(1984).

/2/ L.A.Zadeh, Fuzzy Set, Information & Control, 8(1965).