Pansystems Analysis of Generalized Fuzzy Network

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Let G be a given set, W be a mathematical system. If $g \in W \uparrow G^2$, g is called W-binary relation of G.

Pansystems Reduce Principle[1]: Let $g \subset G^2 \times W$, $D \subset W$, then $g \circ D \subset G^2$, $G = \bigcup G_i \, (d\, \delta(f))$, where δ is a pansystems operator, $f \in \{(g \circ D \vee I)^a, \ (g \circ D)^b\}$ $a, b = (n), t, n = 0, 1, \cdots$.

The principle provides more than one hundred theorems to dispose of generalized fuzzy network. The generalized fuzzy network is a network with panweight. Pansystems network analysis is the generalized fuzzy network studied by viewpoint and method of pansystems methodology. It mainly investigates series—parallel analysis, clustering— discupling, shengke automaton network. In this paper, the so—called flow is discussed by viewpoint of pansystems methodology.

At first, we introduce the concepts of the quasi—opti semi—field and θ—composition.

Definition 1[1] (θ-composition) Let $\theta_1 \in W \uparrow P(W)$, $\theta_2 \in W \uparrow W^2$. $\theta = (\theta_1, \theta_2)$: $(W \uparrow G^2)^2 \rightarrow W \uparrow G^2$, $(g_1 \theta g_2)(x, y) = \theta_1 \{ g_1(x, t) \theta_2 g_2(t, y) | t \in G \}$.

Definition 2 If θ_1, θ_2 satisfy the following conditions, (W, θ_2, θ_1) is called a quasi-opti-semi—field.

1 association and distribution law .

2 $\theta_1(a, b) = a$ or $\theta_1(a, b) = b$.

3 there exists $0 \in W$, $\theta_2(a, 0) = a$ for any $a \in W$.

If $f, g \in W \uparrow G^2$, $\theta_1(f(x, y), g(x, y)) = g(x, y)$, denote $f \leq g$. Let $F = \{ f \,|\, g_1 \leq f \leq g_2, \ f_0 \bar{\theta} f(x, x) = f \bar{\theta} f_0(x, x), \ f \in W \uparrow G^2 \}$ where $f_0 \in \{0\} \uparrow G^2$, $g_i \in W \uparrow G^2$ $i = 1, 2$. $x \in W$, $f \bar{\theta} g(x, y) = \theta_2 \{ f(x, t) \theta_2 g(t, y) \,|\, t \in G \}$ $f \in F$ is called a panweight flow of network (G, g_1, g_2).

Theorem 1 $f \in W \uparrow G^2$ is a panweight flow if and only if $g_1 \leq f \leq g_2$ and $f \bar{\theta} f_0 = f_0 \bar{\theta} f^{-1}$.

Theorem 2 If $F \neq \phi$, then $g_1 \leq f_0 \bar{\theta} g_2^m$, $g_1 \leq g_2^m \bar{\theta} f_0$ for any integer m, where $g^m = g \bar{\theta} g^{m-1}$. and θ_1, θ_2 satisfy the condition: For $a, b, c \in W$ with $\theta_1(a_i, b_i) = b_i$, $i = 1, 2$, then $\theta_1(\theta_2(a_1, a_2), \theta_2(b_1, b_2)) = \theta_2(b_1, b_2)$.

For network (G, g_1, g_2), let $G = \{s\} \cup G_1$, $g_1(x, s) = 0$, for any $x \in G_1$, and define $V \in W \uparrow F$, $V(f) = f \bar{\theta} f_0 (s, s)$,

$F_{f_1} = \{ f \mid V(f_1) \neq V(f)$ and $\theta_1(V(f), V(f_1)) = V(f_1), f \in F \}$

$F^*_{f_1} = \{ l \mid l \neq V(f\ )$ and $\theta_1(l, V(f\ )) \neq l \in V(F) \}$, where $f_1 \in F$.

Theorem 3 $|F_{f_1}| \geqslant |F^*_{f_1}|$, $V(F_{f_1}) = F^*_{f_1}$.

From the equivalence corresponding principle[1], $V \circ V^{-1} \in E[F)$.

Theorem 4 If $f_1 \in F_{i_0} \subset F(d(V \circ V^{-1}))$ such that $|F_{f_1}| = 0$, then (i) $\bigcup F_f$ $(f \in F_{i_0}) = \phi$, (ii) $F_{i_0} \subset F_f$ for any $f \in F - F_{i_0}$.

Let $V^* \in W \uparrow Q$ be induced by V, $V(f_i) = V^*(F_i)$, where $Q = F/_{V \circ V^{-1}}$ $f_i \in F_i$. Now define a binary relation $H \in P(Q^2)$, $(F_i, F_j) \in H$ if and only if $\theta_1(V^*(F_i), V^*(F_j)) = V^*(F_j)$.

Theorem 5 (i) $H \in L_5(Q)$, (ii) $(F_i, F_j) \in H$ if and only if $F_i \subset F$, where $f_i \in F_i$, $f_j \in F_j$.

Pansystems reduce principle plays an important role in treating with series—parallel, clustering—discupling. We apply it to pansystems network analysis.

Let $(N, +, V)$ be a integer quasi-opti-semi-field, where $aVb = \max\{a, b\}$. Network (G, g_1, g_2), where $g_1 \in \{o\} \uparrow G^2$, s, t be source and sink respectively. f be a flow. Define

$N_{f_1} = \{ n \mid$ there exists $(x, y) \in g_2 \circ n$, such that $(x, y) \circ g_2 > (x, y) \circ f \}$.

Theorem 6 If $G_i \subset G$ $(d\, \mathcal{E}_1(f^{+}_{*}))$, $f_* = g_2 \circ N$, then f is a maximal flow if and only if $(s, t) \bar{\in} G_i$ for any i.

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