Some Work on the Pansystematization and Fuzzy Analysis of Scott's Neighbourhood Theory of Computation

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The present paper makes an attempt at describing the essential work of Scott's neighbourhood theory of computation under the framework of pansystems theory and fuzzy theory . It mainly treats the concept of approximable mappings . By the pansystems composition operator , we obtain the formulae of the relation among neighbourhood system , domain and approximable mappings ; prove that the potencies of set of all approximable mappings and the domain images being in are equal ; a hard simulation is not an approximable mapping generally . In addition , we propose a criterion of existence of solutions of a composition relation equation in the class of all approximable mappings .

In symbol $\widetilde{D}$ denotes the domain of neighbourhoods set D over Δ . Write $AM(D_1,D_2)$ for the collection of all fuzzy relations which are approximable mappings between D_1 and D_2 . $AM(D,D)$ is abbreviated to $AM(D)$. Define , for $X \in D_1$ and $f \in AM(D_1,D_2)$, $X \circ f \triangleq \{Y \mid \mu_f(X,Y) = 1\}$.

Theorem 1 . $f \in AM(D_1,D_2)$ iff

i) $X \circ f \in \widetilde{D_2}$ for any X in D_1;

ii) $I_{D_1} \circ f \subseteq f$.

Easily to show that $I_D \in AM(D) \cap R[D] \cap T[D]$; and $I_D \in E[D]$ iff $D = \{\Delta\}$. $I(D) \overline{\in} AM(D)$ when $D \neq \{\Delta\}$. f is an E_s-simulation iff $Y \circ I_D \subseteq (f \circ Y) \circ f$ for any Y in D . $f \in (D_2 \uparrow D_1) \cap AM(D_1,D_2)$ iff $f = D_1 \times \{\Delta_2\}$.

Theorem 2 . (i) Let $f \in AM(D)$. Then

$$f \in R[D] \text{ iff } I_D \subseteq f ;$$

(ii) $f \in S[D] \cap AM(D)$ iff $f = D^2$.

Theorem 3 . If there exists δ in $E[\Delta_1)$ such that $D_1 - \{\Delta_1\} = \Delta_1/\delta$, then $f \in AM(D_1,D_2)$ iff

i) $\Delta_1 \circ f \neq \emptyset$;

ii) $f^{-1} \circ \delta \circ f = \bigcup H(X \circ f)\ (X \in D_1)$;

iii) $I_{D_1} \circ f \circ I_{D_2} \subseteq f$.

Theorem 4 . If $^2 f$ is a hard simulation over D ,

(i) $f \in AM(D)$ when $\|f \circ f^{-1}\| = 1$; and

(ii) $f \bar{\in} AM(D)$ otherwise .

Theorem 5 . If $f \in R[D]$, $f^{(n)} \bigcup I_D \in AM(D)$, then

$$f^t \in AM(D) .$$

In the following we assume that for any f in $AM(D_1, D_2)$, $\mu_f \subseteq \{0,1\}$.

Theorem 6 . $\bigcup f_i \in AM(D)$, if $D^2 = \mathcal{E}_1(I_D)$ and $f_i \in AM(D)$.

Theorem 7 . Let δ , f be in $AM(D_1, D_3)$ and $AM(D_1, D_2)$ respectively . If $\delta = f \circ g$ for some g , then there must exist g' in $AM(D_2, D_3)$ such that $\delta = f \circ g'$.

Theorem 8 . There is g in $AM(D_2, D_3)$ such that $\delta = f \circ g$ iff $\forall (X,Y) \in \delta\ \exists Z \in X \circ f((\bar{\delta} \circ Y \times \{Z\}) \cap f = \emptyset)$.

Theorem 9 . Let $f \in AM(D)$. Then there exists g ($\neq I_D$) fulfilling the following :

i) $g \in AM(D)$;

ii) $g \in T[D] \cap R[D]$

such that

$$f = f \circ g .$$

Theorem 10 . Define $A \bar{\circ} B = \{a \circ b \mid a \in A , b \in B\}$. Then for any D_1 and D_2

(i) $\widetilde{D_2} = D_1 \bar{\circ} AM(D_1, D_2)$;

(ii) $\{D_1\} = AM(D_1, D_2) \bar{\circ} \widetilde{D_2}$;

(iii) $AM(D_1, D_2) = \{D_1 \times x \mid x \in \widetilde{D_2}\}$.

Theorem 11 . $|AM(D_1, D_2)| = |\widetilde{D_2}|$.

References

/1/ Dana Scott , Lectures on a Mathematical Theory of Computation , Oxford University Press , 1981 .

/2/ Wu Xuemou , Pansystems Methodology : Concepts , Theorems and Applications (V-VI), Science Exploration, 4(1983), 1(1984).