Fixed Pansystems Theorems and Fuzzy Fixed Point

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Fixed pansystems theorems is a class of the theorems and investigations about relative stability, which is a kind of typical cases of pansymmetry. In paper[1] , pansystems methodology studied fixed subset, gradul change and sudden change. In paper [2], the authors studied the existence, structure and the number of fixed subsets. This paper continues the researches in paper [1,2], and studies the existence of fuzzy fixed point. Both fixed subset and fuzzy fixed point are the extension of traditional fixed point. In our discussion, we abandon many complex restrictions, and use pansystems relations and operators to study fixed subset and fuzzy fixed point.

At first, we give certain appointment. Let G be a given classical set, $f: G^2 \to [0,1]$, and $\mathcal{F}(G)$ be the collection of all fuzzy subsets contained in G. If $A \in \mathcal{F}(G)$, denote $A \circ f = \{(x,\lambda) \mid \lambda = \bigvee_{y \in G} (A(y) \wedge f(y,x)),\ x \in G\}$. For A, $B \in \mathcal{F}(G)$, define $A \times B = \{(x,y,\lambda) \mid \lambda = A(x) \wedge B(y)\}$. It is clear that for $f,\ g: G^2 \to [0,1]$, $A \in \mathcal{F}(G)$, $(A \circ f) \circ g = A \circ (f \circ g)$. Suppose $\{A_n\}$ is a fuzzy set sequence and $\{f_n\}$ is a fuzzy relation sequence, if $\bigvee_{k=1}^{\infty} \bigwedge_{n=k}^{\infty} A_n = \bigwedge_{k=1}^{\infty} \bigvee_{n=k}^{\infty} A_n = A$, $\bigvee_{k=1}^{\infty} \bigwedge_{n=k}^{\infty} f_n = \bigwedge_{k=1}^{\infty} \bigvee_{n=k}^{\infty} f_n = f$, then $\{A_n\}$ and $\{f_n\}$ are called convergence, and A, f denote $\lim A_n$, $\lim f_n$, respectively. Having given these preliminaries, we begin our discussion.

Definition 1. Let $f: G^2 \to [0,1]$, $A \in \mathcal{F}(G)$, $A \neq \phi$. If $A \circ f = A$, then A is called a fuzzy fixed point of f. When $f: G^2 \to \{0,1\}$ and $A \in P(G)$, then A is called a fixed subset of f .

Theorem 1. If $f: G^2 \to [0,1]$, G is finite, then there exist i, $k \in N$, $i \neq k$, such that $f^{(i+k)} = f^{(i)}$.

Theorem 2. If G is finite, then the necessary and sufficient condition that $\{f^{(n)}\}$ is convergent is that there exists $i \in N$ such that $f^{(i)} = f^{(i+1)}$.

Theorem 3. If G is finite, $\{f^{(n)}\}$ is convergent and $\lim f^{(n)} \neq \phi$, then f has a fuzzy fixed point.

Theorem 4. If G is finite, and there exists n ($n \geqslant i$, i is the same as in theorem 1) such that $f^{(n)} \neq \phi$, then f has a fuzzy fixed point.

Theorem 5. If $\{A \circ f^{(n)}\}$ is convergent, $A_1 = \lim A \circ f^{(n)}$, then $A_1 \circ f \subset A_1$.

Theorem 6. If $f: G^2 \longrightarrow [0,1]$, $A_1 \in \mathfrak{F}(G)$, and $\{A_1 \circ f^{(n)}\}$ is monotonic, then $\{A_1 \circ f^{(n)}\}$ is convergent. Let $A_2 = \lim A_1 \circ f^{(n)}$, then $A_2 \circ f = A_2$.

Definition 2. If $f: G^2 \longrightarrow [0,1]$, denote $\delta_1(f) = (f \vee f^{-1} \vee f^{(0)})^t$. Let $A_i = \max \{A \mid A \in \mathfrak{F}(G),\ A^2 \subset \delta_1(f)\}$, denote $A_i \subset G(d\ \ \delta_1(f))$.

Theorem 7. If $A_i \subset G(d\ \ \delta_1(f))$, then $\{A_i \circ f^{(n)}\}$ is convergent. Let $A_i' = \lim A_i \circ f^{(n)}$, then $A_i' = A_i' \circ f$.

Theorem 8. If $f^t \wedge f^{(0)} \neq \phi$, then f has a fuzzy fixed point. If G is finite, and f has a fuzzy fixed point, then $f^t \wedge f^{(0)} \neq \phi$.

References

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