Some New Researches on Pansystems Methodology and Its Applications to Combinatorics and Generalized Fuzzy Clustering

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Pansystems Methodology (PM) developed a number of concrete research branches and obtained hundreds of mathematical theorems and propositions. Among these concrete researches, pansystems semi-equivalence partition gained much attention and many properties of it is revealed, also the relation between pansystems semi-equivalence partition and other mathematical systems have been discussed. In this paper, we give a brief survey on the researches about pansystems semi-equivalence partition and its application to combinatorics and a kind of generalized fuzzy clistering.

PM uses semi-equivalence relation to deal with a kind of undistinct partition in which two different blocks may possibly intersect each other. All those blocks determined by a semi-equivalence relation forms a quotient system which is called pansystems semi-equivalence partition. A pansystems semi-equivalence partition is in fact a special kind of block design. The necessary and sufficient condition is obtained under which a block design forms a semi-equivalence partition. It is proved that semi-equivalence relations are conservative under the following pansystems operations: confinement, projection, embodiment, conjunction, disjunction, etc. It is shown that for a n-elements set G, there exists a pansystems semi-equivalence partition of G which has more than $(m+1)^r m^{k-r}$ blocks where m, k, r are integers satisfying the equation $n = km+r$ and $0 \leq r < k$.

By discussing the relation between a pansystems semi-equivalence relation and its complement, it is shown that a pansystems semi-equivalence partition may not be a König system, meanwhile a König system may not be a semi-equivalence partition. The relation between Konig systems and pansystems semi-equivalence partitions is discussed and the necessary and sufficient condition is obtained under which a semi-equivalence partition is surely a Konig system.

The above discussion leads to a series extension of Dilworth theorem since Dilworth theorem represents obviously a relation between a partial order relation and its complement.

Under the framework of pansystems relations transformation, the black-box principle, state space method, white-box and grey-box in cybenertics are extended to pan-box principle and pansystems observocontroliblity. Meanwhile, PM developed new abstract network analysis and discoupling principle, extended Burnside theorem in cybernetics and combination clarifying principle in communication. By using $\mathcal{E}_i$ operators, it is determined that how much information can be transmitted without confusion, and it is suggested that we can enlarge the information that can be carried through embody combination.

Another part of PM research is to investigate pansystems logic conservation under various kind of transformation, especially, the structure relative conservation under the combinatorial embodiment. Under this framework, PM developed pan-weighted network analysis, obtained pan-system operation principle and the assiociative law of panweighted network, discovered the relation between the optimal orders of a group of subdecisions and the optimal order of whole decision.

In the investigation of a special form of pansymmetry, we studied certain properties of the covering number N(k, m, n). The covering number N(k, m, n) is defined as follows:

If S is a set of n elements, N(k, m, n) is the smalleast number of m-subsets of S with the property that every k-subset of S is contained in one of these m-subsets.

Concerning covering number N(k, m, n), many authors have studied it and obtained a lot of results. Y.Shiloach and U.Vishkin, S.Zaks evaluated the value of $N(2, m, n)$ for all $m \geqslant \frac{1}{2}n$; M.K.Fort Jr and others determined the value of $N(2, 3, n)$; W.H.Mills evaluated $N(3, 4, n)$ for $n \not\equiv 7 \pmod{12}$. We extended these results and determined the value of $N(3,m,n)$ for $3m \geqslant 2n$ which is shown by the following two theorems.

Theorem A. If $3m > 2n$, then $N(3, m, n)$ is the function of m/n only and

$$N(3, m, n)=\begin{cases} 4, & 3/4 \leqslant m/n < 1; \\ 5, & 5/7 \leqslant m/n < 3/4; \\ 6, & 2/3 < m/n < 5/7. \end{cases}$$

Theorem B. If $3m = 2n$, then $N(3, m, n)$ is not the function of m/n only and

$$N(3, m, n)=\begin{cases} 6, & m = 4k, \ n = 6k; \\ 7, & m = 4k-2, \ n=6k-3. \end{cases}$$

For general integers k, m, n, it is proved that if $(k+t)m < kn$, then $N(k, m, n) \geqslant k+t+1$. Furthermore, it is proved that if $(k+1)m \geqslant kn$, $m < n$, then the above equality holds, i.e. $N(k, m, n) = k+1$.

We deduced areccurebce formula of $N(k, m, n)$ as follows:

$$N(k, m, n) \geqslant n/m \cdot N(k-1, m-1, n-1),$$

and this gives a lower bound of $N(k, m, n)$:

$$N(k, m, n) \geqslant \frac{n!\,(m-k-1)!}{m!\,(n-k-1)!} \cdot \left\{\frac{n-k-1}{m-k-1}\right\}.$$

The value of $N(k, m, n)$ is also determined if $m/n \geqslant 2k-1/2k+1$ as follows:

Theorem C. If $m/n \geqslant 2k-1/2k+1$, then $N(k, m, n)$ is the function of m/n only and

$$N(k, m, n)=\begin{cases} k+1, & k/k+1 \leqslant m/n < 1; \\ k+2, & 2k-1/2k+1 \leqslant m/n < k/k+1. \end{cases}$$

Theorem D. If $m/n < 2k-1/2k+1$, then $N(k, m, n) > k+2$.

Theorem E. If $k = 4$, we have

$$N(4, 6, 8) = 7, \quad N(4, 9, 12) = 8.$$

This theorem shows that if $k = 4$ and $m/n < 2k-1/2k+1$, then $N(k, m, n)$ is not the function of m/n only.

Pansystems semi-equivalence partition is related not only to block design in cimbinatorics but also to the so called fuzzy clustering. Fuzzy clustering is to partition a group of elements according to a fuzzy relation on this elements set G. Usually, a fuzzy relation on G is a mapping f from G^2 to the interval $[0,1]$. The final partition of G is determined by a binary relation δ (usually a equivalence relation) on G which is obtained by choosing a certain number $\lambda \in [0,1]$, and let δ be the maximal subset of G^2 such that $f(\delta) \subset [\lambda,1]$.

Pansystems methodology extended this kind of fuzzy clustering by using pansystems simulating relations. Let G, W be two nonempty sets, let f be a pansystems simulating relation between G^2 and W, i.e. $f \subset G^2 \times W$, then for any subset D of W, we can obtain a binary relation $f \circ D$ on G. Based on this relation, the pansystems semi-equivalence partition of G can be determined by applying $\mathcal{E}_i$ operators. Here the set W needs not to be the interval or a lattice. By choosing different subsets of W, we can obtain many pansystems semi-equivalence partitions of G according to just one simulating relation.

For any given simulating relation $f \subset G \times F$, we can also determine a partition of G and a partition of F as follows: $G = \bigcup G_i(df)$, $F = \bigcup F_i(df)$, here $F_i \times G_i = \max\{A \times B: A \times B \subset f\}$. The relation between this kind of partition and pansystems semi-equivalence partition is discussed, and it is proved that if $G' = G \cup F$ and $f'(\subset G'^2) = f \cup G^2 \cup F^2$, then

$$\{F_i : i=1, 2, \cdots, m\} = \{D \cap F: D \subset F(d\,\mathcal{E}_1(f'))\} - \{F\},$$
$$\{G_i : i=1, 2, \cdots, m\} = \{E \cap G: E \subset G(d\,\mathcal{E}_1(f'))\} - \{G\}.$$

References

1. Wu Xuemou, Pansystems Methodology: Concepts, Theorems and Applications (V-VI), Science Exploration. 4(1983), 1(1984).

2. Zhu Xuding and Wu Xuemou, Some New Researches on Pansystems Methodology and Combinatorics, (to appear).