On the Problem of Equilibrium under Fuzzy Preferences: Applications of Fuzzy Sets and Pansystems Methodology to Economics

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[illegible] this paper we substitute preferences in mathematical economics and discuss the centre problem of equilibrium[4] of it in [illegible] pansystems methodology[1].

1. Some Assumptions

(1) Consumption Set is a aggregation of goods which consumers [illegible]. Given n kinds of goods, consumption set is often taken as [illegible] quadrant R_+^n of n-dimentional Euclid space.

(2) Fuzzy Preferences are judgment on superior of consumers [illegible]. They are quasi-similar fuzzy relations[3] on consumption [illegible]. [illegible] R be a quasi-similar fuzzy relation. $\mu_R(x, y)$ denotes [illegible] level that commodity x is better than commodity y.

(3) Price Standar Form is a set

$$P_n=\left\{p \,\middle|\, p=(p_1, \ldots, p_n),\ p_j \geq 0,\ 1 \leq j \leq n,\ \sum_{j=1}^{n} p_j = 1\right\}.$$

2. Expression of Model

[illegible] a pure exchange economy, given consumption set G, l consumers, [illegible] fuzzy preference R_i and i-th initial resources as

$$\xi^i=(\xi_1^i, \ldots, \xi_n^i) \geq 0 .$$

Then social resources are

$$\xi=(\xi_1, \ldots, \xi_n)=\sum_{i=1}^{l}\xi^i > 0 .$$

[illegible] there is a price $\hat{p} \in P_n$ such that i-th demand of goods $\hat{x}^i \in G$ [illegible]

(1) Pareto Optimal Principle: $\forall i(1 \leq i \leq l)$, there exists not $[illegible] \in$ [illegible] that $\hat{p}x \leq p\xi^i$ and $\mu_{R_i}(x, \hat{x}^i) > \mu_{R_i}(\hat{x}^i, x)$.

(ii) Balence of Supply and Demand (Walras Law): $\sum_{i=1}^{l}\hat{x}^i \leq \sum_{i=1}^{l}\xi^i$.

3. Main Conclusions

Definition. Let R be a fuzzy preference on consumption set G. $\forall x, y, z \in G$ and $0<\alpha<1$, if $\mu_R(x, z) \geqslant \mu_R(z, x)$ and $\mu_R(y, z) \geqslant \mu_R(z, y)$ imply $\mu_R(\alpha x+(1-\alpha)y, z) \geqslant \mu_R(z, \alpha x+(1-\alpha)y)$, then R [illegible] convex. Again $\forall x^k, y^k, x, y \in G$, $k=1, 2, \ldots$, if

$\rightarrow$ [illegible] $(k \rightarrow \infty)$ and $y^k \rightarrow y (k \rightarrow \infty)$ and $\mu_R(x^k, y^k) \geqslant \mu_R(y^k, x^k)$ imply $\mu_R(x, y) \geqslant \mu_R(y, x)$, then R is said to be continuous.

Theorem. If consumption set G is compact and ξ is an inner [illegible] G , every i-th fuzzy preference is concave and continuous, [illegible] solution $[\hat{x}^1, \ldots, \hat{x}^l; \hat{p}]$ of equilibrium for the [illegible]

[illegible]. Outline of Proof of the Theorem

[illegible]) Relations between Prices and Demand of Goods: $\varphi_i \subset P_n \times G$, [illegible] $G = \{x \mid 0 \leqslant x \leqslant a,\ a = (a_1, \ldots, a_n) > \xi\}$.

(2) Relation between Prices and Supply of Glut Goods: $\psi \subset P_n \times F$, [illegible] $F = \xi - E$, $E = \{x \mid 0 \leqslant x \leqslant la\}$.

[illegible]) Relation between Goods and Minimum Price: $\mu \subset G \times P_n$.

[illegible]) Existence of Equilibrium (Constitution of K-Relation): $f \subset$ [illegible], where $(p, x) \circ f = x \circ \mu \times p \circ \psi \ (\forall (p, x) \in P_n \times E)$. By Kukutani Theorem[5] , there is a fixed point $(\tilde{p}, \tilde{x}) \in (\tilde{p}, \tilde{x}) \circ f$. Then $\hat{p} = \tilde{p}$ and $\hat{x} = \xi - \tilde{x}$ [illegible] balanced price and demand of goods respectively.

[illegible]. Computation of Balanced Price and Demand

$$d((p, x), (p, x) \circ f) = \sqrt{d^2(p, x \circ \mu) + d^2(x, p \circ \psi)} ,$$

$$d(p, x \circ \mu) = \min_{q \in x \circ \mu} d(p, q) \quad \& \quad d(x, p \circ \psi) = \min_{y \in p \circ \psi} d(x, y) .$$

[illegible] problem is to minimize

$$U = d((p, x), (p, x) \circ f)$$

[illegible] in $P_n \times E$ and to find out minimum points. In some special [illegible] can determine the minimum points by Lagrange multiplier [illegible] linear and nonlinear programing or other methods.

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