Pansystems Investigation of Arrow's Problem under Fuzzy Preference

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The problem of social choice from individual values is a major topic in mathematical economics. The well-known Arrow's impossibility theorem[1] states that there is no constitution which simultaneously satisfies Axioms of Completeness and Transitivity and the conditions of Free Orderings, Weak Pareto Principle, Independence of Irrelevant Alternatives and Nondictatorship. From the viewpoint of pansystems methodology[2], the problem of social choice is just to seek for a body-shadow relation which keeps pansystems symmetry of Arrow's conditions as could as possible. Along this course, we have gone further into the topic of [3]. In this summary we try to make another attempt to by-pass Arrow's paradox through fuzzy idea. In fact, human beings are not infinitely discriminative, so their preferences are often fuzzy. According to the practical background we propose following important concept.

<u>Definition.</u> Suppose G is a non-empty set, $g \in P(G^2)$. If g satisfies

(1) $(g - g^{-1})^{(2)} \leq g$

(2) $(g - g^{-1}) \circ (g \wedge g^{-1}) \leq g$

(3) $(g \wedge g^{-1}) \circ (g - g^{-1}) \leq g$,

Then we say g is quasi-transitive.

The notations appeared in above definition could be found their meanings in [2]. We emphasize that quasi-transitivity is [illegible] a new modeling of fuzzy transitivity.

Example. Suppose an individual puts sugar in his coffee. He [illegible] the difference between 2 spoons of sugar and 1.8 [illegible] of sugar, and he is also indifferent 1.8 spoons to 1.6 [illegible] sugar, but he perheps indeed prefers 2 spoons to 1.6 [illegible].

In this example, the man's indifference isn't transitive but [illegible]sitive. In other words, his preference is fuzzy.

Through the introducing of quasi-transitivity and the series [illegible] analysis established in pansystems methodology, we [illegible] following theorem.

Theorem. Let G be a non-empty set, $g_i \in P(G^2)$ satisfying [illegible] Completeness and Quasi-Transitivity, where $i \in I=\{1,2,\cdots,$ $\}$. [illegible] $g = [\bigwedge_{i\in I}(g_i - g_i^{-1})] \vee \{ G^2 - [(\bigwedge_{i\in I}(g_i - g_i^{-1})) \vee (\bigwedge_{i\in I}(g_i - g_i^{-1}))^{-1}]\}$ [illegible] Axioms of Completeness and Quasi-Transitivity and the [illegible] of Free Orderings, Weak Pareto Principle, Independence [illegible] Irrelevant Alternatives and Nondictatorship.

Corollary. If only we weaken the Axiom of Transitivity into [illegible] of Quasi-Transitivity, then rational social choices [illegible].

References

[1] [illegible]rrow, [illegible]cial Choice and Individual Values, New York, [illegible] Wiley & Sons Inc., 1963.

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