ON THE PANSYSTEMS LOGIC ALGORITHMS IN RANK-ORDERING OBJECTS

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1. Introduction

As well known, the problem Rank-ordering the objects under discussion is of a widely useful one. It is involved in a great many fields, such as economics, management science, productive process of industry, psychology, system science, computer science, ... etc.. This paper presents what so called the binary comparison average method used for fuzzy pattern recognition, which is proposed in [2] in terms of pansystem embodiment-seperation-tolorance principle [1] and the binary comparison principle in psychology. It is compared with that in [3], and point out that whatever the method in [2] or that in [3] is a special pansystem logic algorithm.

In the operations for control and recognition, pansystem embodiment can usually increase the freedoms of the operations, thus could strengthen the observocontrol means and augment the selectivity, plasticity, comparability, seperability as well as the compatibility. As a result, it facilitates identifying objects and getting rid of the faults, in such a way, the observocontrol level for analysing things and the adaptivity in running things are increased. This is what so called the concept of seperation-tolorance in operations research. It is a concept of principle in system engineering and large scale systems operations, and also an important scheme of principle in game. such as the following are all of pansystem embodiments: the filterings from lower dimessions to higher ones; the systems from special purpose to general purpose (bus); from lower freedom to higher one, and the transitive closure concept of computer net; from the blackbox to whitebox or panbox, together with the extension of state spaces and the augment of related matrices; the analysis from headrope to net, the generalized codiagnosis, the syntheses of hologram, as well as the investigations of some pansystem logic etc.. the embodiment-seperation-tolorances concept can be related with the binary comparison principle in psychology. In the binary comparison average method of membership function for pattern recognition proposed in [2] the pansystem embodiment-seperation-tolorance principle is used. It amounts to find a special pansystem logic algorithm.

As a matter of convience, we call the binary comparison average method in [2] as A-method, and the binary comparison extreme value method in [3] as E-method. Both are pansystem logic transform algorithms based on the binary comparison principle in psychology. According to the knowledge in psychology, rank-ordering many objects can first determine the comparison level of each couple of objects among them under discussion, and then convert these levels into that of multiobjects through a certain algorithm. Thus, we reduce the problem rank-ordering objects to find a correspondent pansystem logic algorithm for the special problem.

2. The Descriptions of E-method

The E-method in [3] can briefly describle as follows:

(1) First define a pairwise function (or called P-function) as the following:

If $x,y\in X$, then write $f_y(x)$ for the membership function of x, pairwise function or P-function by name, and defining that $f_x(x)=1$ and that $f_y(x)=0$, when $x\bar{\in}X$.

(2) Next, for $x,y\in X$, the fuzzy measurement of choosing x over y is denoted by $f(x/y)$, called a relativity function (or R-function) and is defined as

$$f(x/y)=f_y(x)/max[f_y(x),f_x(y)]\ ,\quad x,y\in X \tag{1}$$

(3) Again, for $x,y\in X$ $(i=1,2,...,n)$ we define

$$f(x/y_1,y_2,\cdots,y_n)\equiv f(x/T)=\min_i f(x/y_i) \tag{2}$$

$$(T=(y_1,y_2,\cdots,y_n))$$

representing the fuzzy measurement of choosing x over all the y_i. Because $f(x_j/x_j)=1$, we have that if $x_i\in X$ (i 1,2,...,n), then

$$f(x_j/T)=\min_i[f(x_j/x_i)] \tag{3}$$

where $T=(x_1,x_2,...,x_n)$. Formula (1) together with (3) present the E-method in [3].

3. The Descriptions of A-method and Some Illustrations

The A-method in [2] is an alternative which is more clear and simple in the description of concept than E-method in [3]. It is a transform algorithm based on the pansystem embodiment-projection principle [1], stating as follows:

To find the membership function of a fuzzy set $f^*: G \to [0,1]$, we can first find out according to the binary comparison principle the membership function of binary fuzzy set $\varsigma: G \times H \to [0,1]$, and then convert ς to f^* through a certain appropriate algorithm. without loss of generality, now let H be G, then $\varsigma: G^2 \to [0,1]$, basing on the established measure $\sigma(y)$, we define the relative function as

$$f^*(x/T) = \int_T \varsigma(x,y)\, d\sigma(y) \qquad (4)$$

normalizing $\int_T \varsigma(x,y)\, d\sigma(y) = 1$ then can consider $f^*(x/T)$ as the membership function of T; where the measure $\sigma(y)$ plays the role of weight parameter, and the integration is a certain generalized mean. We point out here that functions $\varsigma(x,y)$, $\varsigma(y,x)$ in [2] correspond to $f_y(x)$, $f_x(y)$ in [3] respectively. While, in case of equal weight, formula (4) become

$$f^*(x/T) = \frac{1}{n} \sum_{i=1}^{n} \varsigma(x, y_i), \quad y_i \in T \qquad (5)$$

Where n is the number of elements of T.

Several illustrations are in order:

Example 1. Let G be the set of beautiful flowers, in case of existing cherry blossoms ($=x$) and chrysanthemums ($=y$), and if the beautiful degree of cherry blossoms is 0.8, and 0.7 the chrysanthemums, then we denote them as $g(x,y)=0.8$, $g(y,x)=0.7$ respectively. When taking account of cherry blossoms ($=x$) and dandelions ($=z$), it may be asserted that $g(x,z)=0.9$, $g(z,x)=0.5$, $g(y,z)=0.8$, $g(z,y)=0.4$.

For the sake of comparison, although the example is the same as in [3], but conceptually, we look upon f (x) in [3] as the membership function of fuzzy relation $g(x,y)$. It seems to be more nature and generalized to do so. In general, if $y \neq z$, then mostly $g(x,y) \neq g(x,z)$. For the convience to treat mathematically, we take $g(x,x)=1$.

So far we can compute the relative function. If the judgements above have equal weights, then get the following:

$$f(x/(x,y,z)) = \frac{1}{3}(g(x,x) \;\; g(x,y) \;\; g(x,z)) = 0.9,$$
$$f(y/(x,y,z)) = \frac{1}{3}(g(y,x) \;\; g(y,y) \;\; g(y,z)) = 0.83,$$
$$f(z/(x,y,z)) = \frac{1}{3}(g(z,x) \;\; g(z,y) \;\; g(z,z)) = 0.63,$$

$f(x/(y,z))=\frac{1}{2}(g(x,y)\quad g(x,z))=0.85,$

$f(y/(y,z))=\frac{1}{2}(g(y,y)\quad g(y,z))=0.9,$ and so on.

It can be used as a standard for rank-ordering objects, when we interpreting $f(x\ T)$ as the average membership to T. For example, cherry blossoms is more beautiful than chrysanthemums and so is chrysanthemums than dandelions when $T=(x,y,z)$. If $T=(y,z)$, then chrysanthemums is more beautiful than cherry blossoms when T has many members, and so it is reasonable to regard the values computing out from f as a criterion for rank-ordering objects.

Example 2. In the above exaple, if g is presented with unequal weights, for example, $\sigma(x)=\frac{1}{10}$, $\sigma(y)=\frac{8}{10}$, $\sigma(z)=\frac{1}{10}$, then for $T=(x,y,z)$, we have $f(x/T)=\frac{1}{10}(g(x,x)+8g(x,y)+g(x,z))$ $=0.83$, $f(y/T)=0.95$, $f(z/T)=0.47$. At the present case, chrysanthemums is more beautiful than cherry blossoms, and cherry blossoms is more beautiful than dandelions.

When $\sigma(x)=\frac{1}{20}$, $\sigma(y)=\frac{1}{20}$, $\sigma(z)=\frac{18}{20}$, then we have $f(x/T)=0.9$, $f(y/T)=0.8, f(z/T)=0.945$. According to the weights like these, the beautiful degrees of these three flowers arrange in decreasing order as dandelions, cherry blossoms and chrysanthemums.

Example 3. We consider the eldest son $(=x_1)$, the secondary son$(=x_2)$, the third son$(=x_3)$ and their father $(=z)$ as the objects pattern. If we only take account of the problem of the eldest son and the seeondary son resembling their father, then the degree of the eldest son resembling his father is 0.8, that of the seeondary son resembling his father is 0.5. If only take account of the secondary son and the third son, then the degree of the secondary son resembling his father is 0.4, that of the third son resembling his father is 0.7. If only take account of the eldest son and the third son, then the degree of the eldest son resembling his father is 0.5, that of the third is 0.3. In that case, we construct the binary fuzzy relation as follows:

$$g(x_1,x_1)=1,\quad g(x_1,x_2)=0.8,\quad g(x_1,x_3)=0.5;$$
$$g(x_2,x_1)=0.5,\quad g(x_2,x_1)=1,\quad g(x_2,x_1)=0.4;$$
$$g(x_3,x_1)=0.3,\quad g(x_3,x_2)=0.7,\quad g(x_3,x_3)=1;$$

in case of equal weigt, $\sigma(x_1)=\sigma(x_2)=\sigma(x_3)=\frac{1}{3}$, then for $T=(x_1,x_2,x_3)$ holds

$$f(x_1/T)=0.766,\quad f(x_2/T)=0.633,\quad f(x_3/T)=0.666.$$

These values of f determine the order of the eldest son, the secondary son and the third son resembling third father. But in literature [3], with a still more complicate algorithm offered $f(x_1/T:z)=1$, $f(x_2/T:Z)=\frac{4}{7}$, $f(x_3/T:z)=\frac{3}{5}$, and the conclusion (the order of resemble) is the same.

Example 4. Corresponding to the experiment 1 in [3], the average $g(x_i, x_j)$ of the relative beautiful degree to five sorts of flowers x_i, i 1,2,...5, judged by 10 experimenters, is

$g(x_1, *)$: 1, 0.64, 0.42, 0.60, 0.38;
$g(x_2, *)$: 0.62, 1, 0.58, 0.68, 0.64;
$g(x_3, *)$: 0,82, 0.84, 1, 0.82, 0.76;
$g(x_4, *)$: 0.28, 0.34, 0.34, 1 0.32;
$g(x_5, *)$: 0.60, 0.58, 0.50, 0.70, 1.

We obtained by A-method that

$$f(x_1/T)=0.608,\ f(x_2/T)=0.704,\ f(x_3/T)=0.848,$$
$$f(x_4/T)=0.456,\ f(x_5/T)=0.676.$$

and the correspondent values obtained by E-method is 0.51, 0.69, 1.00, 0.46, 0.66. Evidently, the conclusions resulted by both algorithms is the same: $x_3 > x_2 > x_5 > x_1 > x_4$.

Example 5. Corresponding to the experiment 2 in [3], let's take the unknown pictures as the standard ones, and the typical pictures in the different groups of pictures as x_i, and further to study the pattern recognition problem through considering that the unknown pictures resemble which of the typical pictures. Now supose that we are to make a judgement of the unknown pictures resembling which of the script letters a, b and c respectively, and the averages answered by 10 experimenters are

$g_1(x_i, x_j)$			$g_2(x_i, x_j)$			$g_3(x_i, x_j)$		
1,	0.32,	0.54	1,	0.50,	0.58	1,	0.60,	0.56
0.46,	1,	0.40	0.64,	1,	0.70	0.34,	1,	0.52
0.46,	0.64,	1	0.32,	0.38,	1	0.42,	0.52,	1

where $g_k(x_i, x_j)$ is the degree of x_i resembling the unknown picture k when the judgement contains x_i. By using the A-method we obtain

$$f_1(x_1/T)=0.62,\ f_1(x_2/T)=0.62,\ f_1(x_3/T)=0.70;$$
$$f_2(x_1/T)=0.69,\ f_2(x_2/T)=0.78,\ f_2(x_3/T)=0.56;$$
$$f_3(x_1/T)=0.72,\ f_3(x_2/T)=0.62,\ f_3(x_3/T)=0.65.$$

Thus the picture 1 is identified as $x_3(=c)$, picture 2 as x_2 $(=b)$, picture 3 as $x_1(=a)$. The conclusion obtained here is the same as that in [3].

4. Several Equivalence Theorems to A-method and E-method

In this section, let's investigate the equivalence between formula (3) to E-method and formula (5) to A-method. For the sake of convience, let

$$\Sigma_j = \sum_{\substack{p=1\\ p\neq i}} \zeta(x_j, x_p),$$

$$M_j = \zeta(x_j, x_i) = \min\{\zeta(x_j, x_1), \cdots, \zeta(x_j, x_n)\},$$

$$m_j = \min_p f(x_j/x_p), \; p = 1, 2, \cdots, n;$$

then we can easily prove that:

Lemma 1. If $(\Sigma_j - \Sigma_k)$, $(M_j - M_k)$ and $(m_j - m_k)$ possess the same sign or vanish simultaneously when $j \neq k$, then $f(x_j/T)$, $f(x_k/T)$ and $f^*(x_j/T)$, $f^*(x_k/T)$ have the same order.

We can prove, in terms of mathematical induction and lemma 1, that:

Theorem 1. If $(\Sigma_j - \Sigma_k)$, $(M_j - M_k)$ and $(m_j - m_k)$ possess the same sign or vanish simultaneously for whatever $j \neq k$ $(j, k = 1, 2, \ldots, n)$, then the E-method formula (3) is equivalent to the A-method formula (5) with equal weight.

Lemma 2. If $f(x_j/T) > f(x_k/T)$ and $f^*(x_j/T) > f^*(x_k/T)$, then $(m_j - m_k)$ and at least one between $(\Sigma_j - \Sigma_k)$ and $(M_j - M_k)$ are greater than zero simultaneously.

Lemma 3. If $f(x_j/T) < f(x_k/T)$ and $f^*(x_j/T) < f^*(x_k/T)$, then $(m_j - m_k)$ and at least one between $(\Sigma_j - \Sigma_k)$ and $(M_j - M_k)$ are less than zero simultaneouly.

Lemma 4. If $f(x_j/T) = f(x_k/T)$ and $f^*(x_j/T) = f^*(x_k/T)$, then either $(m_j - m_k)$, $(\Sigma_j - \Sigma_k)$ and $(M_j - M_k)$ vanish simultaneously, or $m_j - m_k = 0$ and $(\Sigma_j - \Sigma_k) = -(M_j - M_k)$.

It follows from lemma 2 to lemma 4 that:

Theorem 2. If the E-method formula (3) is equivalent to the A-method formula (5) with equal weight, then it is necessary to hold one of the following conditions:

(N). $(m_j - m_k)$ and at least one between $(\Sigma_j - \Sigma_k)$ and (M -M) possess the same sign;

(N). either $(m_j - m_k)$, $(\Sigma_j - \Sigma_k)$ and $(M_j - M_k)$ vanish simultaneously, or $m_j - m_k = 0$ and $(\Sigma_j - \Sigma_k) = -(M_j - M_k)$.

It is demonstrated by the illustrations in section 3 that the case with unequal weight is not equivalent to that with equal weight in A-method. Thus it follows by the argument in the present section that: E-method is equivalent to special case in A-method. And we see further through the following examples that: A-method is more reasonable and has a wider range of aplications than E-method.

We may verify one by one that all the illustrations in section 3 satisties the conditions in theorem 1 each. In the following, we shall use the example 4 (denoting the example 1 for it) as an illustration.

Example 1. The average $g(x_i, x_j)$ of the relative beautiful degree to five sorts of flower x_i, $i=1,2,\ldots,5$, judged by 10 experimenters is

$$\begin{array}{lccccc}
\varsigma(x_1, *): & 1, & 0.64, & 0.42, & 0.60, & 0.38; \\
\varsigma(x_2, *): & 0.62, & 1, & 0.58, & 0.68, & 0.64; \\
\varsigma(x_3, *): & 0.82, & 0.84, & 1, & 0.82, & 0.74; \\
\varsigma(x_4, *): & 0.28, & 0.34, & 0.34, & 1, & 0.32; \\
\varsigma(x_5, *): & 0.68, & 0.58, & 0.50, & 0.70, & 1.
\end{array}$$

known by the section 3, from A-method $f^*(x_1/T)=0.608$, $f^*(x_2/T)=0.704$, $f^*(x_3/T)=0.848$, $f^*(x_4/T)=0.456$, $f^*(x_5/T)=0.676$; and the same order determining by both algorithms is $x_3 > x_2 > x_5 > x_1 > x_4$. We can immediately verify that the example satisfies the conditions of theorem 1.

Example 2. The values of $\varsigma(x_i, x_j)$ in example 1 are left unchanged except $\varsigma(x_2, *)$ taking the following values:

$$\varsigma(x_2, *): \quad 0.62,\ 1,\ 0.37,\ 0.68,\ 0.85;$$

here, $f^*(x_i/T)$ $(i=1,2,\ldots,5)$ computed by A-method are the same as example 1. But the correspondent values by E-method are 0.51, 0.44, 1, 0.41, 0.66, thus it is given by E-method that: $x_3 > x_5 > x_1 > x_2 > x_4$. Evidently, in this example, E-method is not equivalent to A-method. We point out that, in this example, the necessary conditions of both algorithms being equivalent is satisfied (i.e. the negative and inverse theorem of theorem 1 is satisfied). But the sufficient conditions of that is not satisfied (i.e. the negative and inverse theorem of theorem 2 is not satisfied).

Example 3. The set of ten students x_i $(i=1,2,\ldots,10)$ is denoted by T. Suppose that, in an enterance examination for college, the $\varsigma(x_i, x_j)$ obtained by comparing in pairs the higher and the lower of marks in total is

(x ,): 1, 1, 0, 1, 0, 1, 1, 0, 0, 0;
(x ,): 0, 1, 0, 0, 0, 0, 0, 0, 0, 0;
(x ,): 1, 1, 1, 1, 0, 1, 1, 0, 0, 0;
(x ,): 0, 1, 0, 1, 0, 0, 0, 0, 0, 0;
(x ,): 1, 1, 1, 1, 1, 1, 1, 0, 0, 0;
(x ,): 0, 1, 0, 1, 0, 1, 1, 0, 0, 0;
(x ,): 0, 1, 0, 1, 0, 0, 1, 0, 0, 0;
(x ,): 1, 1, 1, 1, 1, 1, 1, 1, 1, 1;
(x ,): 1, 1, 1, 1, 1, 1, 1, 1, 0, 1;
(x ,): 1, 1, 1, 1, 1, 1, 1, 0, 0, 1;

Actually computing by A-method, we obtain the values of $f^*(x_i/T)$ $(i=1,2,\ldots,10)$: $\frac{5}{10}, \frac{1}{10}, \frac{6}{10}, \frac{2}{10}, \frac{7}{10}, \frac{4}{10}, \frac{3}{10}, 1, \frac{9}{10}, \frac{8}{10}$. the correspondent rank-ordering is: $x_8 > x_9 > x_{10} > x_5 > x_3 > x_1 > x_6 > x_7 > x_4 > x_2$; but the correspondent values of $f(x_i/T)$ by E-method are: 0,0,0,0,0,0,0,1,0,0. The correspondent rank-ordering is: $x_8 > x_1 = x_2 = \cdots = x_{10}$. We can verify that, for this example, the necessary condition of equivalence for both algorithms in theorem 2 are not satisfied. This example shows that, the rank-ordering of objects can't specified by E-method can be determined completely by A-method. Thus it can be seen that A-method is more useful and reasonable than E-method.

References

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