# FUZZY LOGIC IMPLEMENTATION IN THE REALIZATION OF FUZZY ALGORITHMS

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## Abstract

An approach to implement the evaluation of the fuzzy logical predicates by a composition of binary and multivalued logical operators is developed. This approach is used to formulate a model to the realization of the fuzzy conditional statement and fuzzy programs in general. A set of typical operators would be specified to cover the execution of each of fuzzy operators. Some interpretative examples are given to demonstrate the approach and the model of realization.

## INTRODUCTION

In the papers by Mamdani [ 6 ], Chang [ 2 ] and in others [ 1, 7, 9, 12 ], many models in the execution of fuzzy programs were demonstrated, by using, as a formal basis, the compositional rule of inference and the notions of fuzzy sets [12, 13]. In our point of view, the most constructive means to formulate the models of fuzzy programs executions should be the logical. However, in spite of the vast publications in the implementation of fuzzy logic [3, 4, 5, 8], it is difficult to refer to a constructive approach, which allows to develope the formal logical basis of fuzzy programs execution.

In this paper we would develop an approach to implement the evaluation of fuzzy predicates executions by a composition of multivalued and binary logical operators. This approach would be applied to formulite the execution of the fuzzy conditional statement. On this basis a set of typical operators in specified to cover the execution of fuzzy algorithms.

## 1. FUZZY OPERATORS : THEIR FORMAL REPRESENTATION AND EXECUTION

By the fuzzy operator we mean any instruction to execute some action with data given in fuzzy linguistic terms and could be represented as some fuzzy sets. Let us consider the fuzzy conditional statement ( F.C.S.), which could be used as a basic operator in the construction of fuzzy programs:

$$\text{IF} \quad Q \quad \text{THEN} \quad F_s \quad \text{ELSE} \quad F_t \tag{1.1}$$

where: Q - is some fuzzy predicate; $F_s$, $F_t$ - are some fuzzy functional operators. The F.C.S. requires to examin the execution of Q-predicate and the transfering to execute $F_s$ or $F_t$, when: Q is satisfied or not satisfied, respectively.

We are concerned in the formal description of Q- predicates and F- operators in the following two levels:

- their formal representation in fuzzy algorithms and programs ;
- their formal model of execution in the processing of fuzzy information. We would formalize the following Q-predicates and F-operators:

1. $Q_r$ - the fuzzy predicate to verify the execution of some fuzzy resemblance relation.

<u>The formal representation</u>:

$$Q_r \; ::= \; \langle \, LV \;\; \text{is} \;\; T_i \, \rangle \quad , \tag{1.2}$$

where: LV- is some linguistic variable, defined on the universe of discourse $U_{LV}$ ; $T_i$- the fuzzy term value of LV, defined as some fuzzy subset of $U_{LV}$ :

$$T_i = \{ d_j , \mu_{ij} \} \quad , \tag{1.3}$$

$d_j \in U_{LV}$ and $\mu_{ij} \in [0,1]$.

<u>The formal model of execution</u>:

Let: E (LV) - denotes an operator which assigns the linguistic variable, LV , some exact value, $d' \in U_{LV}$ i.e.:

$$E ( LV ) \; ::= \; \langle \, LV := d' \, \rangle \tag{1.4}$$

Applying E (LV) to (1.2), the execution of $Q_r$ could be formalized as following:

$$E ( Q_r ) \; ::= \; \langle \, d' \;\; \text{is} \;\; T_i \, \rangle \tag{1.5}$$

Let: T( Q ) - denotes the truth degree of Q- predicate execution, evaluated on the truth interval [0,1]. Then, $T(Q_r)$ could be evaluated as in (1.6) :

$$T ( Q_r ) = \mu_{ij} \qquad (1.6)$$

Example. Let: LV = CW = THE CHANGE IN THE PATIENT WEIGHT ; $U_{CW} = [-10,+10]$ Kg ; $T_i$ = HIGHLY INCREASED = { ( 2,0.3) ; (5,0.6); ( 8,0.8) ; (10,1) } . Suppose that : $E_1$( CW) = 2 Kg and $E_2$ ( CW) = =8 Kg . Then, T ( $Q_r$) could be evaluated as following:

$$T ( Q_{r1} ) = \mu_{i1} = 0.3$$

$$T ( Q_{r2} ) = \mu_{i3} = 0.8$$

2. Qo - the fuzzy predicate with the fuzzy ordered relation.

The formal representation:

$$Qo_1 \; ::= \langle \; LV \;\; T_{Ro} \;\; LV \; \rangle \qquad (1.7)$$

$$Qo_2 \; ::= \langle \; LV \;\; T_{Ro} \;\; T_i \; \rangle , \qquad (1.8)$$

where: $T_{Ro}$- denote the fuzzy term of the ordered relation $R_o$, such as : "APPROXIMATELY EQUAL","SLIGHTLY LESS THEN" and so on.

Observing (1.7) and (1.8), it would be clear that $T_{Ro}$ - is some fuzzy set of the ordered pairs ( $d_i$, $d_j$ ) $\in U_{LV} \times U_{LV}$.

To simplify the evaluation of Qo - execution we would represent $T_{Ro}$ - by using the uni-dimensional support of the elements: $d_{ij} = d_i - d_j$ , $i \neq j$,

$$T_{Ro} = \{ d_{ij}, \; \mu_{T_{Ro}} (d_{ij}) \} \qquad (1.9)$$

The formal execution:

Let: $E_1$( LV) = $d_i'$ and $E_2$( LV ) = $d_j'$. Then, the execution of $Q_o$-predicate, E( $Q_o$), could be formalized as following:

$$E( Q_{o1} ) \; ::= \langle d_i' \;\; T_{Ro} \;\; d_j' \rangle \qquad (1.10)$$

$$E( Qo_2 ) \; ::= \langle d_i' \;\; T_{Ro} \;\; T_i \rangle \qquad (1.11)$$

The truth degrees of T( Qo ) - could be evaliated as following:

$$T( Qo_1 ) = \mu_{T_{Ro}} ( d_{ij}' ) \qquad (1.12)$$

$$T( Qo_2 ) = \mathrm{Min} ( \mu_{T_{Ro}} ( d_{ij}' ), \mu_{T_i} ( d_i' ) ), \qquad (1.13)$$

where: $d_{ij}' = d_i' - d_j'$.

Example. Consider the following linguistic variable, [ 10 ] : LV = FT = " THE FERMENTATOR TEMPERATURE STATE " , defined on $U_{FT}$ = [ 10, 35 ] C°. Let in (1.7) : $E_1$(FT) = NORM = 25 C°;

$E_c(FT)$ = 20 C° and $T_{Ro}$ = MUCH MORE THAN = $\{(2, 0.2); (5, 0.6); (8, 0.9)\}$. Then:

$$d'_{ij} = 25 - 20 = 5 \; C^{o} \; ;$$

$$T(Q_o) = \mu_{ij}(5) = 0,6 \; .$$

3. $F_A$ - The Fuzzy Assignment Operator.

<u>The formal representation</u>:

$$F_A \; : : = \; L_f \; : = \; T_{fi} \; , \qquad (1.14)$$

where: $L_f$ - some linguistic variable of F - operator, defined on $U_{Lf}$; $T_f$ - the i-th fuzzy term value of $L_f$, defined as some fuzzy set

$$T_{fi} = \left\langle d_{fj}, \; {}_i( d_{fj} ) \right\rangle , \qquad (1.15)$$

$d_{fj}$ $U_{LF}$.

Example. $L_f$ : = " HIGH VALUE " .

<u>The formal model of execution</u>:

let $E(F_A)$ - denotes the execution process of $F_A$-operator, whith could be formalized:

$$E(F_A) \; : : = \left\langle L_f \; : = \; d'_f \right\rangle , \qquad (1.16)$$

where: $d'_f \in U_{Lf}$ - is some element of the support of $T_{fi}$, whith could be determined by applying the infered value of $\mu_i(d'_f)$ as an address to $T_{fi}$ in (1.15). The execution of F-operators requires to realize some rule of inference whith should infer $\mu_i(d'_j)$ in dependence on the T(Q) - the truth degree of Q-predicate execution.

4. $F_M$ - The Fuzzy Mathematical Operator.

<u>The formal representation</u>:

$$F_M \; : : = \left\langle L_f \; : = \; L_f \; * \; T_{fi} \right\rangle , \qquad (1.17)$$

where: * -denotes some typical mathematical operator.

Example. $F_M$ = " INCREASE $L_f$ SLIGHTLY "

$L_f$ : = $L_f$ + " SLIGHT VALUE " .

<u>The formal model of execution</u>:

the execution of $F_M$- operator, $E(F_M)$, could be formalized as following:

$$E(F_M) \; : : = \left\langle L_f \; : = d_f \; * \; d'_{fi} \right\rangle , \qquad (1.18)$$

where: $d'_{fi} \in U_{Lf}$- some element of the support of $T_{fi}$, retrieved by the relative infered value of $\mu_i(d'_{fi})$.

The above formalized interpretation to the representation and execution of Q - predicates and F-operators would be used as a basis to formulate a constructive model to the execution of F.C.S. and in general the fuzzy programs.

## 2. A CONSTRUCTIVE EVALUATION OF THE Q- PREDICATES EXECUTION.

In (1.6), (1.12) and (1.13) we have demonstrated the evaluation of $T(Q_r)$, $T(Qo_1)$ and $T(Qo_2)$, respectively, by using the concept of the truth degree of compotibility of fuzzy sets, $\mu_{Ti}(d_j) \in [0,1]$. As: T(Q) - could be assigned infinite values of the truth interval [0,1], then the suggested technique does not allow to organize the constructive procedures to the evaluation of Q-predicates and control transfering to F-operators.

We shall develope an approach to evaluate the execution of Q-predicates by applying a composition of binary and multivalued logical operators, introduced for this purpose.

Let the truth interval [0,1] be divided in a finite set of truth levels $\gamma$, i.e. :

$$\gamma = \quad a_1, \ldots, a_i, \ldots, a_k \qquad 0,1 \quad , \tag{2.1}$$

with $a_1 = 0$ and $a_k = 1$.

Let: $T_m(Q)$ - denotes an operator, which assigns Q some truth value from set , i.e. :

$$T_m \quad : \quad Q \longrightarrow \gamma \tag{2.2}$$

Assume that: $T_m(Q_i)=T_{mi}$. By applying $T_m$ to the set of fuzzy predicates $\{Q_i, Q_j\}$, the set of the variables, $\{T_{mi}, T_{mj}\}$, taking values from $\gamma$ would be recieved. As : $\gamma$ is a finite set of truth levels, then: $T_{mi}$ and $T_{mj}$ are multivalued logical variables and the following logical operations ( $\&$, $\vee$, $\neg$ ) could be defined as following:

$$\begin{aligned} &\text{i)} \quad T_{mi} \,\&\, T_{mj} \;:=\; \mathrm{Min}\,(T_{mi}, T_{mj}), \\ &\text{ii)} \quad T_{mi} \vee T_{mj} \;:=\; \mathrm{Max}\,(T_{mi}, T_{mj}), \\ &\text{iii)} \quad \neg T_{mi} = (a_k - T_{mi}) = (1 - T_{mi}) \end{aligned} \tag{2.3}$$

Further we shall call $T_m(Q)$:The Multivalued Operator of the Q-predicate evaluation.

Let: $\hat{A}$-denotes some fix truth level in $\gamma$ and call it:The Threshold of fuzzy thruth. It shold be noticed that , $\hat{A}$ , can take any value in $\gamma$ unless $a_1$ and $a_k$, i.e. :

$$\hat{A} \in \{a_2, \ldots, a_i, \ldots, a_{k-1}\} \tag{2.4}$$

We shall say that the fuzzy predicate, Q , is fuzzily true ($\hat{t}$); if $T_m(Q) \geqslant \hat{A}$ and fuzzily false ($\hat{f}$) , if $T_m(Q) < \hat{A}$.

Let: $T_b(Q)$ - denotes an operator, which assigns the execution of Q-predicate some truth value of the set: $\{\hat{f}, \hat{t}\}$ or $\{0,1\}$, respectively, as following:

$$T_b(Q) = \begin{cases} 1 \, , & \text{if } T_m(Q) \geqslant \hat{A} \\ 0 \, , & \text{if } T_m(Q) < \hat{A} \end{cases} \tag{2.5}$$

Assume that $T_b(Q_i) = T_{bi}$. By applying $T_b(Q)$ to the set: $\{Q_i, Q_j\}$ the set of the variables $T_{bi}$ and $T_{bj}$, all taking values from $\{0,1\}$, would be recieved. Then : $T_{bi}$ and $T_{bj}$ are some binary variables and the binary logical operations, $\otimes$ , could be defined with them as in their traditional definition, where:

$$T_{bi} \otimes T_{bj} \qquad 0,1 \tag{2.6}$$

We shall call $T_b(Q)$- the binary operator of Q- predicate evaluation. The above introduced $T_m(Q)$ and $T_b(Q)$- operators would be applied in the model formulation to execute the fuzzy conditional statement.

## 3. A FORMAL MODEL TO THE EXECUTION OF FUZZY CONDITIONAL STATEMENT.

In order to formulate the model of F.C.S. execution we would require that each fuzzy term : $T_q$ and $T_f$ of the Q-predicates and F-operators should be represented with their degrees of compatibility are some truth levels in $\gamma$, i.e.:

$$T_q = \{d_{qj} \, , \; a_{qj}\} \quad ; \tag{3.1}$$

$$T_f = \{d_{fj} \, , \; a_{fj}\} \quad , \tag{3.2}$$

where: $d_{qj} \in U_{Lq}$, $d_{fj} \in U_{Lf}$ ; $a_{qj}, a_{fj} \in \gamma$ .

We need , also, to introduce the following accessing operators:

a) $A(d_{qj})$ - an operator, which applys $d_{qj}$ as an address to $T_q$ of (3.1) to retrieve the value of compatibility $a_{qj} \in \gamma$ , i.e. :

$$A(d_{qj}) = a_{qj} \tag{2.3}$$

F. $A(a_{fj})$ - an operator, which applys $a_{fj}$ as an address to $T_f$ of (3.2) to retrieve the relative element of the support $d_{fj} \in U_{Lf}$, i.e.:

$$A\ (\ a_{fj}\ )\ =\ d_{fj} \tag{3.4}$$

The model of F.C.S. execution would be decomposed in the following three procedures:

i - the procedure of Q-predicate evaluation ( Q - procedure ),

ii - the procedure of control the transfer to F-operators ( C - procedure ).

iii - the procedure of F-operator execution ( F - procedure ).

We would spesify a set of typical operators to organize the realization of each of these procedures.

1. The organization of Q - procedure.

By this procedure the execution of Q - predicate would be evaluated by the operators $T_m(Q)$ and $T_b(Q)$ and could be organized as following :

1.1. Execute $E(L_q)$:

$$L_q\ :=\ d'_q$$

1.2. Execute $A(d'_q)$:

$$A(d'_q)\ =\ a'_q$$

1.3. Evaluate $T_m(Q)$:

$$T_m(Q)\ :=\ a'_q$$

1.4. Evaluate $T_b(Q)$:

$$T_b(Q) := \begin{cases} 1\ , & \text{if } T_m(Q) \geqslant \hat{A} \\ 0\ , & \text{if } T_m(Q) < \hat{A} \end{cases}$$

Observing the above formalized steps, we could specify a set of typical operators to the realization of Q - procedure.

Assertion 1. The set of typical operators: $Q_T = \{ E(L_q), A(d_q),$ $:=, T_m(Q):\hat{A} \}$ is sufficient to organize the realization of Q - procedure.

2. The organization of C - procedure.

The C - procedure should control the transfer to execute $F_s$, when: Q - is executable ($T_b(Q) = 1$) or to execute $F_t$, when Q - is unexecutable ($T_b(Q) = 0$). We would introduce the following control operator $C(T_b)$:

$$C(T_b) \longrightarrow F_s \ , \quad \text{if } T_b(Q) = 1 \ ,$$
$$C(T_b) \longrightarrow F_t \ , \quad \text{if } T_b(Q) = 0 \ , \qquad (3.5)$$

where: $\longrightarrow$ - denotes the control of transfering to F - operators.

The execution of C - procedure could be organized by 2.1.

2.1. Execute C $(T_b)$.

Assertion 2. The set of typical operators $C_T = \{C(T_b), \Omega\}$, where: $\Omega$ - denotes the unconditional operator of transfer is sufficient to organize the execution of C - procedure.

3. The organization of F - procedure.

By the execution of F - procedure, the linguistic variable, $L_f$, should be assigned some exact element, $d_f^! \in U_{Lf}$, of the support of $T_f$. In order to elect the element of the support, $d_f^!$, the relative value of the truth degree of compatibility, $a_f^! \in \gamma$, should be defined and the applied as an address to retrieve $d_f^!$. Then: F - procedure should include some rule of inference, to infer the relative value $a_f^!$ in accordance to the truth degree of Q - predicate execution, $T_m(Q) = a_q^!$. In general, the F.C.S. could be formalized as a composition of two logical implications :

$$Q \Longrightarrow F_s \qquad (3.6)$$
$$\neg Q \Longrightarrow F_t \qquad (3.7)$$

Then, the rules of inference in the execution of F.C.S. could be formalized as following:

$$\frac{Q \Longrightarrow F_s}{a_q' \Longrightarrow a_{fs}^! = ?} \qquad (3.8)$$

$$\frac{\neg Q \Longrightarrow F_t}{a_q' \Longrightarrow a_{ft}^! = ?} \qquad (3.9)$$

In [2,6,7], the compositional rule of inference suggested in [13] was applied to realize the inference procedure. But the application of this rule requires to execute the time consuming operators of Max and Min on the big massive of fuzzy relations ( n-dimensional matrix). We would develope an approach to organize the inference procedure, based on the fact that the truth degree of the concequent could not exceed that of the insident in the evaluation of the logical implications. Applying this approach, the truth degree : $a_{fs}^!$ and $a'_{ft}$

of (3.8) and (3.9) could be evaluated as following:

$$a'_{fs} \;:=\; T_m(Q) \;=\; a'_q \;, \qquad (3.10)$$

$$a'_{ft} \;:=\; \neg T_m(Q) \;=\; 1 - a'_q, \qquad (3.11)$$

As $a'_{fs}$ and $a'_{ft}$ are defined, then the relevant elements of the support $d'_{fs}$ and $d'_{ft}$ could be elected by the accessing operator: $A(a'_f)$ formalized in (3.4). Upon $d'_{fs}$ or $d'_{ft}$ the functional operators $F_A$ or $F_M$ would be realized by applying the typical operators of their model of execution of (1.16) and (1.17), respectively. Then F - procedure could be organized by the following steps.

3.1. Evaluate:

$$a'_{fs} = a'_q \;, \quad \text{if} \quad T_b(Q) = 1$$
$$a'_{ft} = 1 - a'_q \;, \quad \text{if} \quad T_b(Q) = 0$$

3.2. Execute $A(a'_f)$:

$$d'_{fs} \;:=\; A(a'_{fs})$$
$$d'_{ft} \;:=\; A(a'_{ft})$$

3.3. Execute $F_A$ and $F_M$ - operators.

Assertion 3. The set of typical operators $F_t = \{ :=, *, 1 - T_m(Q), A(a'_f) \}$ - is sufficient to organize the execution of F - procedure.

We would demonstrate the above formulated procedures in the execution of F.C.S. of the fuzzy algorithm to control the fermentation process of antibiotics [ 10 ] .

Example.

"IF $\underbrace{FT = HIGH}_{Q}$ THEN $\underbrace{FP := LOW}_{F_{As}}$ ELSE $\underbrace{FP := MEDIUM"}_{F_{At}}$ ,

where: FT = "THE FERMENTATOR TEMPERATURE STATE", $U_{FT}$ = 10, 35 C°, FP = "THE FERMENTATOR PRESSURE STATE", $U_{FP}$ = [ 0.2, 3 ] atm;

$$HIGH = \{ (15;\ 0.4),\ (25;\ 0.8),\ (32;\ 1) \} \;;$$
$$LOW = \{ (0.4;\ 0.9),\ (1;\ 0.5),\ (2;\ 0.3) \} \;;$$
$$MEDIUM = \{ (1;\ 0.5),\ (1.2;\ 0.6),\ (1.5;\ 0.9) \} \;.$$

Assume that: $\hat{A}$=0.5 and E(FT)=15 C°. Then the execution of above F.C.S. would be :

THE EVALUATION OF $Q_r$- PREDICATES EXECUTION.

Execute $A(d'_q)$:

$$a'_q := A(d'_q) = A(15) = 0.4.$$

1.2. Evaluate $T_m(Q)$:

$$T_m(Q) := a'_q = 0.4.$$

1.3. Evaluate $T_b(Q)$:

$$T_b(Q) := 0, \text{ as: } T_m(Q) = 0.4 \quad \hat{A} = 0.5.$$

2. THE TRANSFER PROCEDURE EXECUTION.

2.1. $C(T_b) = C(0) = F_{At}$

3. THE INFERENCE PROCEDURE EXECUTION.

3.1. Evaluate $a'_{Ft}$ :

$$a'_{Ft} := \neg T_m(Q_r) = 1 - 0.4 = 0.6 ;$$

3.2. Retrieve $d'_{Ft}$ :

$$d'_{Ft} := A(0.6) = 1.2 \text{ atm} ;$$

3.3. Execute $F_{At}$ :

$$FP := 1.2 \text{ atm}.$$

The above formalized procedures could be applied in the execution of fuzzy programs, constructed with the F.C.S., as it is represented in [ 11 ] .

4. CONCLUDING REMARKS

In this paper we have developed an approach to implement the evluation of fuzzy predicates by a composition of multivalued and binary logical operators. A model to the execution of F.C.S. by using the specified, in the paper, tyical operators was formulated. This model could be used as a basis in the organization of fuzzy programs realization. Some important charectarstics could be recognized:

1. The execution of fuzzy operators and programs is controlled by an exect data, $d'_q \in U_{Lq}$ , retrieved to the system by the evaluational operator: E ( Lq ) . In dependence on the outside world exact data the execution of Q- predicate and the transfer to S or t directions of branching are evaluated.

2. The execution of fuzzy operators and programs could be realized by a set of typical operators and specified procedures. This approach could allow to organize some constructive modules in the systems of fuzzy information processing . A step in this direction is demonstrated in [11].

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