THE EQUATIONS OF SOME GEOMETRIC PATTERNS

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Abstract

We advance a new concept about fuzzy sup-inf type linear space, and give the equations of some geometric patterns, and point out an interesting proposition; if a fuzzy linear space contains non-trivial subspaces, its rank be not always less than that of the whole space.

1. Fundamental Concepts

For posing a new concept about fuzzy sup-inf type linear space, we recommend some concepts in the paper(1):

Definition 1.1

A semiring is a set R provided with two binary operations +, × from R×R to R, which satisfy.

(1) $a + b = b + a$,

(2) $a + (b + c) = (a + b) = c$,

(3) $a(bc) = (ab)c$,

(4) $a(b + c) = ab + ac$,

(5) $(b + c)a = ba + ca$.

Definition 1.2

For any commutative semiring R with 0, 1, V_n will denote the set of n-tuples of elements of R. The following operations are defined:

$$(r_1, r_2, \ldots, r_n) + (s_1, s_2, \ldots, s_n) =$$

$$(r_1+s_1, r_2+s_2, \ldots, r_n+s_n)$$

and

$$r(r_1, r_2, \ldots, r_n) = (rr_1, rr_2, \ldots, rr_n).$$

Let 0 denote (0, 0, ... , 0).

The members of V_n have the properties:

(1) $v+w=w+v$,	(5) $a(v+w)=av+aw$,
(2) $v+(w+u)=(v+w)+u$,	(6) $1v=v$,
(3) $(ab)v=a(bv)$,	(7) $v+0=0+v=v$,
(4) $(a+b)v=av+bv$,	(8) $0v=a0=0$.

Definition 1.3 A subspace of V_n is a subset W of V_n such that $0\in W$ and for v, $w\in W$ we have $v+w\in W$. A linear combination of elements of a set S is a finite sum $\Sigma\, a_i s_i$ where $s_i\in S$ and $a_i\in R$. The set of all linear combinations of elements of S is called the span of S, denoted (S). If (S)=W then S is called a spanning set or set of generators for W.

Definition 1.4 A basis for a subspace W is a minimal spanning set for the subspace.

Definition 1.5 A set S of vectors over a semiring R is independent if and only if for no $V\in S$ is a linear combination of elements of $S\setminus\{V\}$. If V is a linear combination of elements of $S \setminus \{V\}$ it is said to be dependent.

Definition 1.6 A basis C over the fuzzy algebra is a standard basis, if and only if whenever $C_i = \Sigma a_{ij} c_j$ for c_i, $c_j \in C$, then $a_{ii} c_i = c_i$.

The fuzzy algebra $[0,1]$ under the operations $a+b=\sup\{a,b\}$ $ab=\inf\{a,b\}$ is commutative semirings with 0,1. We denote it as I.

Now, we pose a new concept about the fuzzy sup-inf type linear space.

Difinition 1.7 If R=I, then V_n is called a fuzzy sup-inf type linear space over a semiring I, and denoted as V_n.

2. The equations of some geometric parttens

From the paper(1), we saw that the span of any subset S of V_n is a subspace of V_n, and is contained in every subspace of V_n which contains S. Any finitely generated subspace has a unique standard basis.

Using the standard basis of any finitely generated subspace (S), we may give a equation of (S) and the boundary curve of (S). Such equations are always unique.

Example 1 Let $S_1 = \{(0.2, 0.8), (0.8, 0.2)\}$, then

$$(S_1) = a_1(0.2, 0.8) + a_2(0.8, 0.2),$$

where

$$0.2 \le a_1, a_2 \le 0.8 .$$

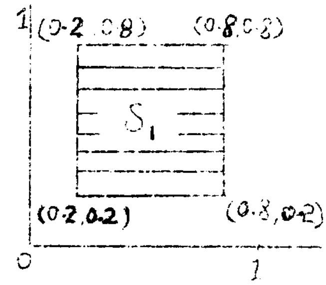
Let B_1 denote a boundary curve of S_1, then

$$B_1 = a_1(0.2,0.8)+a_2(0.8,0.2),$$

where $a=0.8$, $b\leq 0.8$ or $a\leq 0.8$, $b=0.8$

or $a=0.2$, $b\geq 0.2$

or $a\geq 0.2$, $b=0.2$



Example 2 Let $S_2=\{(0.2,0.8),(0.7,0.3)\}$, then

$$(S_2)=a_1(0.2,0.8)+a_2(0.7,0.3),$$

where $0.2\leq a_1,\ a_2\leq 0.8$.

and

$$B_2=a_1(0.2,0.8)+a_2(0.7,0.3),$$

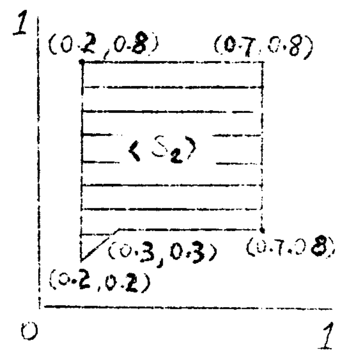
where $a_1=0.8$, $a_2\leq 0.7$ or

$a_1\leq 0.8$, $a_2=0.7$

or $a_1\geq 0.2$, $a_2=0.2$

or $a_1=0.3$ $a_2\geq 0.3$

or $a_1=0.2$, $0.2\leq a_2\leq 0.3$.



Example 3

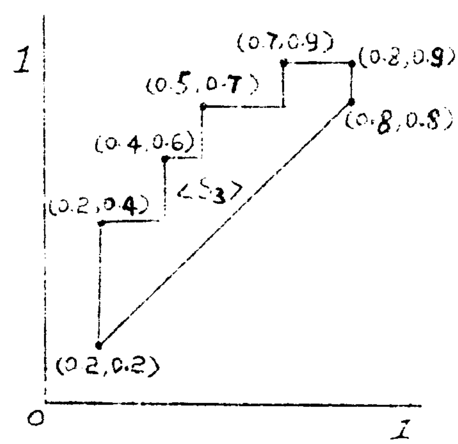
Let $S_3=\{(0.2,0.4),(0.4,0.6),$
$(0.5,0.7),(0.7,0.9),$
$(0.8,0.8)\}$

then

$$(S_3)=a_1(0.2,0.4)+a_2(0.4,0.6)$$
$$+a_3(0.5,0.7)+a_4(0.7,0.9)$$
$$+a_5(0.8,0.8),$$

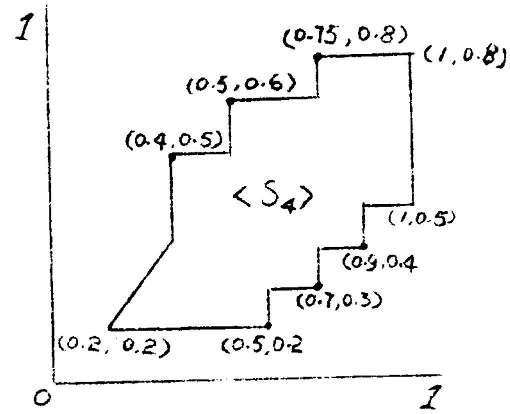
where $0.2\leq a_i\leq 0.9$,

$i=1,2,\ \ldots\ ,5$.



Example 4 Let

$S_4=\{(0.4,0.5),(0.5,0.6),(0.75,0.8),(0.5,0.2),(0.7,0.3),(0.9,0.4),(1,0.5)\}$,



then

$$\langle S_4\rangle=a_1(0.4,0.5)+a_2(0.5,0.6)+a_3(0.75,0.8)+a_4(0.5,0.2)+a_5(0.7,0.3)+a_6(0.9,0.4)+a_7(1,0.5),$$

where $0.2\leq a_i\leq 1$, $i=1,2,\ldots,7$.

3. An interesting proposition

From above examples, we can suggest the following interesting proposition.

Proposition 3.1 If a fuzzy sup-inf type linear space contains non-trivial subspace, its rank need not always be less than that of the whole space.

$$S=\{(r_1,r_2,\ldots,r_n),(r_{n+1},r_{n+2},\ldots,r_{2n}),\ldots\ldots,(r_{mn+1},r_{mn+2},\ldots,r_{(m+1)n})\},$$

Where $r_i>r_j(i>j)$, then we can easily proof that S is independent and that card S=m+1.

Therefore, when $m\geq n$, the cardinal number m of (S) is greates than n of V_n 's.

Corollary 3.2 Card (S)=1, 2, ... , n, ...

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