AN IDENTIFICATION ALGORITHM IN FUZZY RELATIONAL SYSTEMS

W.Pedrycz
Department of Automatic Control & Computer Sci.
Silesian Technical University
44-100 Gliwice,Pstrowskiego 16
Poland

1. INTRODUCTION

A fuzzy system discussed firstly by Zadeh [9] can be treated as a useful and an appropriate tool for processing fuzzy data.Two classes of fuzzy systems are widely discussed,viz.

-fuzzy relational systems(e.g. [3] [5] [6]),

-fuzzy functional systems(mainly fuzzy version of linear systems [4] [7]).

The first class of fuzzy systems deals with a fuzzy relation which describes interrelationships present between input and output of the system.The second one describes situations where the structure of the system under consideration is well known but the collected data are represented as fuzzy sets.A basic problem, which usually appears,is tied with the construction of the model of the fuzzy system.

We shall restrict ourselves to identification problems in fuzzy relational systems. There are some papers devoted to identification problems in fuzzy systems [3] [5] [6] where various techniques are applied to fuzzy and nonfuzzy data.We propose another algorithm for constructing models of fuzzy relational systems where the concept of fuzzy discretization and clustering techniques are applied.The usefulness of the algorithm discussed is demonstrated by solving a numerical example.The paper is organized as follows.In Section 2 we shall review an idea of a fuzzy discretization and clustering techniques in generating a fuzzy partition of the universe of discourse.Next a general scheme of the identification procedure will be considered and some implementation details will be discussed as well. The algorithm is applied to a set of Box-Jenkins data and the results are summarized in Section 4.

2. FUZZY DISCRETIZATION AND FUZZY CLUSTERING ALGORITHMS

The idea of fuzzy discretization introduced in [8] can be sketched as follows.

Let $X_1, X_2, \ldots, X_c$ be basic fuzzy sets defined on the universe of discourse $\mathbf{X}$ ($\mathbf{X} \subset R$), R-real numbers, $X_i : \mathbf{X} \to [0,1]$, $i=1,2,\ldots c$- fulfilling the property,

$$\forall_{x \in R} \; \exists_{1 \le i \le c} \; X_i(x) > 0, \tag{1}$$

viz. every point of $\mathbf{X}$ belongs to at least one of the X_i's-we can say the space $\mathbf{X}$ is "covered" by the family $\{X_i\}_{i=1}^{c}$. It assures that every fuzzy set $X : \mathbf{X} \to [0,1]$ can be expressed in terms of the system of basic fuzzy sets. In order to do so, one calculates the value of a possibility measure of X with respect to X_i, $i=1,2,\ldots,c$ as,

$$p_i = \mathrm{Poss}(X \mid X_i) = \sup_{z \in \mathbf{X}} [\min(X_i(z), X(z))] . \tag{2}$$

Therefore every X can be represented by a sequence of numbers $p_i \in [0,1]$, $i=1,2,\ldots,c$

$$X \to [p_1 \; p_2 \ldots p_c] \tag{3}$$

in which p_i is the value of the possibility measure with respect to X_i obtained with the aid of (2).

If X is a degenerated fuzzy set whose membership function is equal to 1 in exactly one point, say $x_0 \in \mathbf{X}$, and 0 otherwise, Eq. (2) takes the form,

$$p_i = X_i(x_0) . \tag{4}$$

This approach to the approximation of any fuzzy set by means of the basic fuzzy sets has two main advantages, viz.

-reduction the memoryloading in a computer implementation of the algorithms dealing with the fuzzy data;

-the unified treatment of fuzzy and nonfuzzy forms of information.

Nevertheless we should underline the fact that the choice of the basic fuzzy sets and of their number is subjective.They depend on the intuition of the model-builder and therefore can provide some troubles.

Here the fuzzy clustering method can be used for generating the $\{X_i\}_{i=1}^{c}$.

Informally speaking a fuzzy clustering of$\{x_1,x_2,\dots,x_n\}$ being a subset of the elements of into "c" fuzzy clusters is a process of assigning the grades of belongingness of each of the points x_i,i=1,2,...,n to every cluster.The fuzzy clusters can be characterized by a partition matrix $F=[f_{ij}]$,Ii=1,2,...,c, j=1,2,...,n which fulfils the following conditions,

(i) $$\sum_{i=1}^{c} f_{ij}=1, \tag{5}$$

(ii) $$\sum_{j=1}^{n} f_{ij}>0. \tag{6}$$

Every x_j is characterized by means of "c" numbers in the [0,1]-interval.

The membership function of the i-th basic fuzzy set X_i can be obtained by the interpolation of the i-th row of the partition matrix.The most""plausible" number of clusters can be determined by finding the extremalvalue of an index of clustering(e.g. partition coefficient,partition entropy,cf.[1]).

[illegible].THE ALGORITHM FOR CONSTRUCTING THE FUZZY RELATIONAL MODEL OF THE SYSTEM

Bearing in mind the results of the previous section we shall propose the following algorithm leading to the construction of the fuzzy relational model of the system.

1.Collect fuzzy(nonfuzzy) input-output data;

2.Perform clustering of the data and construct the basic fuzzy sets;

3.Express all the data collected in terms of the basic fuzzy sets;

4.Fix the structure of the model;

5.Calculate the fuzzy relation R of the equation minimizing a given performance index ;

6.Test the model obtained against the collected empirical data.

The presented scheme forms the general framework of an identification procedure in fuzzy systems .The details should be specified according to the concrete system under consideration.Some numerical results obtained with this algorithm will be presented in more detail in the next section.

5.NUMERICAL ILLUSTRATION

The identification procedure will be applied to the gas furnace data of Box and Jenkins[2] .This data set is well known and is frequently used as a standard test for identification algorithms.It consists of 296 pairs of input-output observations where the input is the gas flow rate into the furnace and the output is the concentration of CO_2 in the exhaust gases.The sampling interval is equal to 9 seconds.The first step of the identification procedure is to establish the basic fuzzy sets defined on the input and output spaces. The FUZZY C-MEANS [1]was used as the clustering technique and the results of the clustering for $c_i=c_o=5$(c_i and c_o stand for the number of the basic fuzzy sets fixed on the input and output space) are shown in Fig.1.

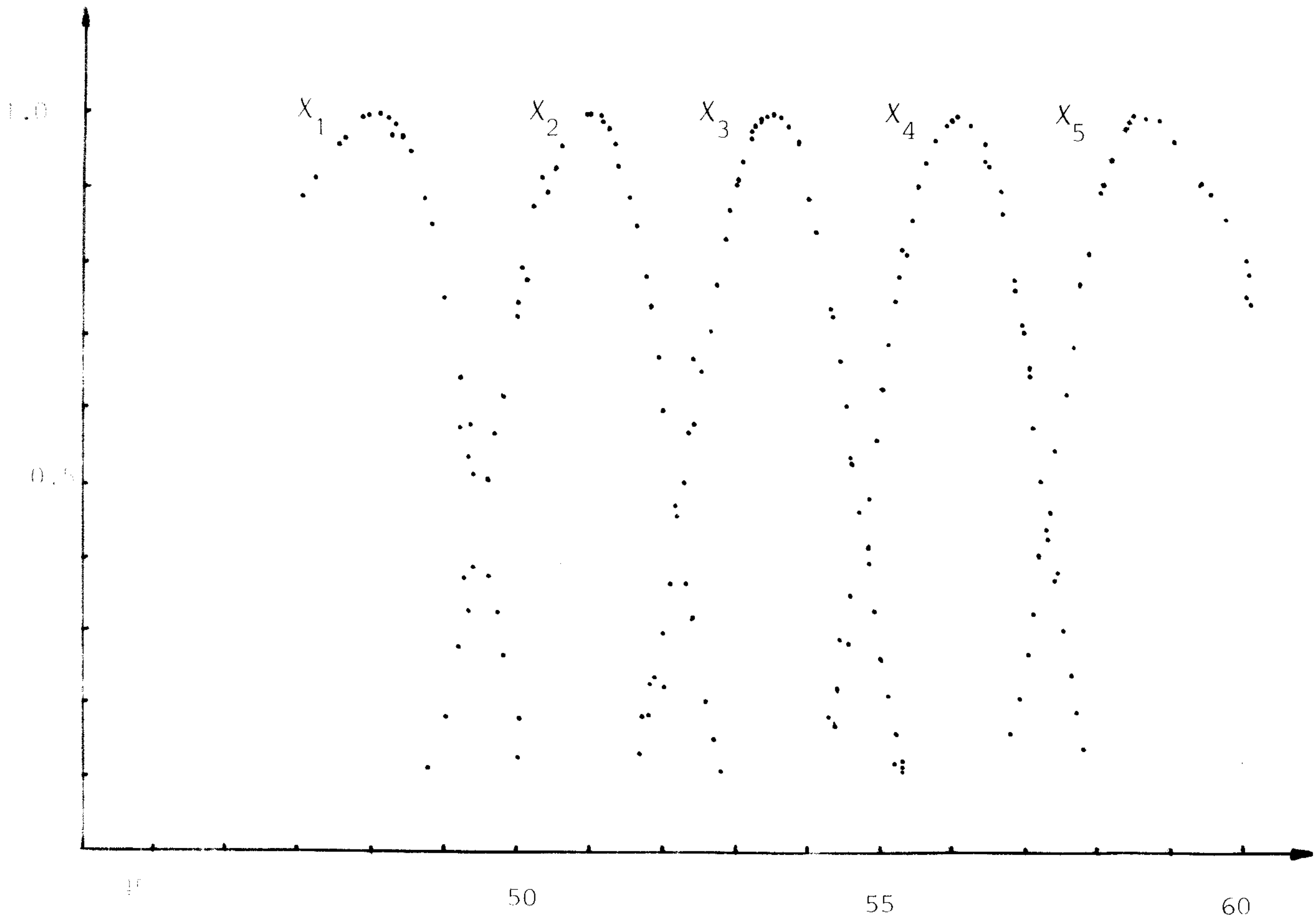

Fig. 1 .The basic fmzzy sets defined on the output space

Two types of fuzzy relational equations are taken into account,

max-min equation of the first order

$$X_{k+1}=U_{k-\tau}\circ X_k\circ R, \tag{7}$$

max-prod equation of the first order

$$X_{k+1}=U_{k-\tau}*X_k*R. \tag{8}$$

$U_{k-\tau}$, X_k, X_{k+1} are fuzzy sets expressed on input and output spaces respectively with the membership functions calculated by means of the basic fuzzy sets; τ stands for the delay($\tau\geq0$) between input and output.

The fuzzy relation R is calculated from the formula,

$$R=\bigcup_{k=\tau+1}^{n-1}(U_{k-\tau}\times X_k\times X_{k+1}), \tag{9}$$

where "×" stands for the cartesian product of the respective fuzzy sets, n denotes the number of elements in the collected data set. The cartesian product is treated according to the composition operator applied and therefore,

-for max-min composition "×" is treated as the minimum operator,

-for max-prod composition "×" is treated as the product operator.

The performance index Q we want to minimize takes the form,

$$Q=\sum_{k=\tau+1}^{n-1}\rho_E(X_{k+1},X_{k+1}), \tag{10}$$

where ρ_E is the Euclidean distance between X_{k+1} calculated via Eq.(7)(or(8)) and X_{k+1} from the collection of fuzzy sets.

The values of Q are displayed in Fig.2.

It is obvious that the max-prod composition operator is preferred with $\tau=2$.

The fuzzy relation R can be treated as a collection of implication statements with the values of the possibility measure assigned to them, e.g.

$$\mathrm{Poss}(\text{if } U_{k-\tau} \text{ is } \mathcal{U}_i \text{ and } X_k \text{ is } \mathcal{X}_j \text{ then } X_{k+1} \text{ is } \mathcal{X}_m)=\lambda_{ijm} \tag{11}$$

The set of linguistic rules (11) can be rewritten in a more compact way in the form of a matrix with the entries,

$$\begin{array}{c|cccc}
 & \mathcal{X}_1 & \mathcal{X}_2 & \cdots & \mathcal{X}_{c_o} \\ \hline
\mathcal{U}_1 & \mathcal{X}_\ell,\lambda_{ij\ell} & & & \\
\mathcal{U}_2 & & \ddots & & \\
\vdots & & & \ddots & \\
\mathcal{U}_{c_i} & & & &
\end{array}$$

where the index ℓ is taken from the relationship,

$$\max_{1\le m\le c_o}\lambda_{ijm}=\lambda_{ij\ell}$$

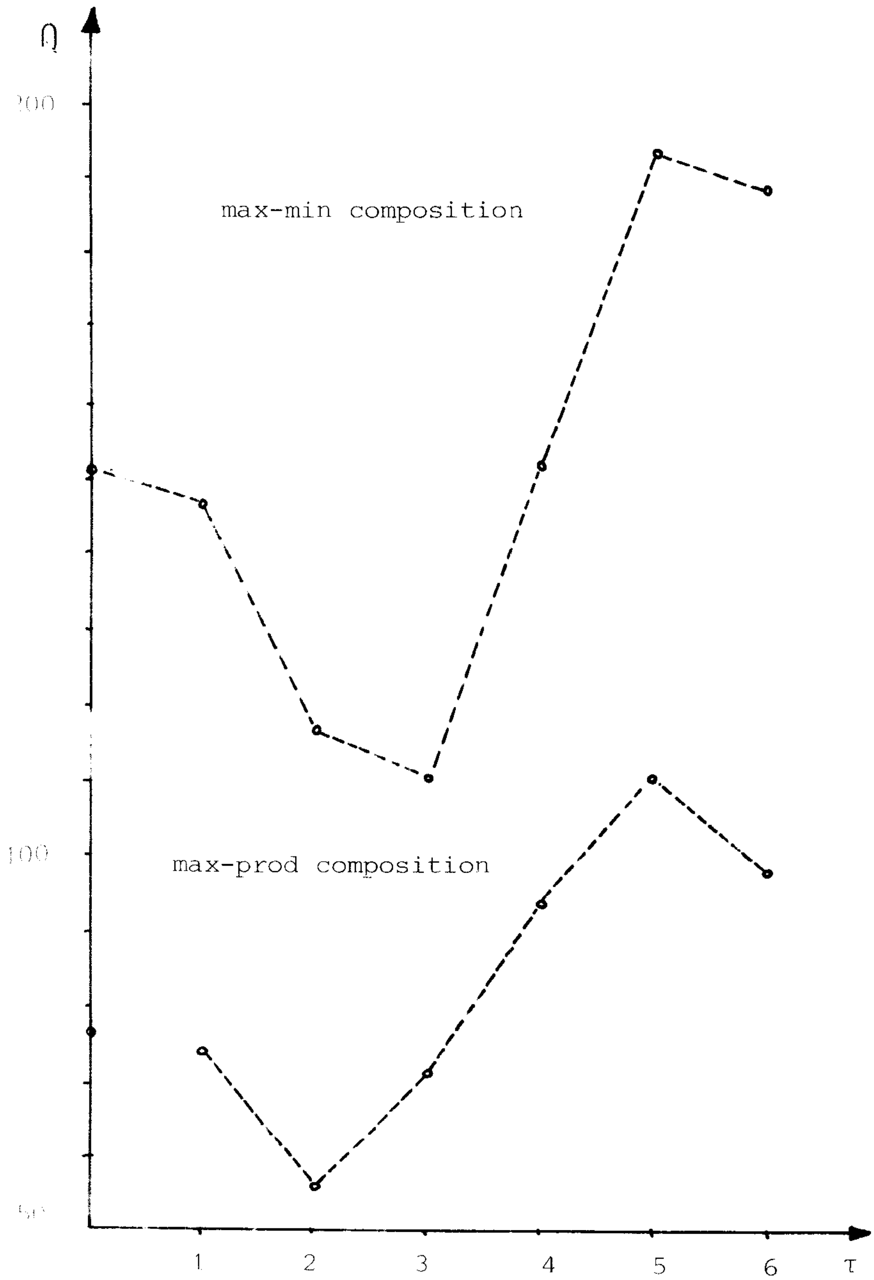

Fig. 1 Ω vs. τ for max-min and max-prod composition operators

The linguistic model derived from the Box-Jenkins data can be summarized in the matrix,

	X_1	X_2	X_3	X_4	X_5
U_1	—	—	X_5 0.92	X_4 0.76	X_5 0.98
U_2	—	X_3 0.73	X_3 0.82	X_4 0.92	X_5 0.95
U_3	X_2 0.49	X_2 0.89	X_3 0.99	X_4 0.98	X_5 0.99
U_4	X_1 0.79	X_2 0.96	X_3 0.81	X_4 0.69	X_5 0.13
U_5	X_1 0.97	X_2 0.74	X_2 0.27	—	—

(— denotes that the greatest value of the measure of possibility does not achieve 0.10).

The use of a centre of gravity method enables us to transform this linguistic description of the system into a numerical description, see Fig.3.

ACKNOWLEDGEMENT

The paper was prepared while the author was on research fellowship at the Department of Mathematics Delft University of Technology, the Netherlands, whose support he gratefully acknowledges.

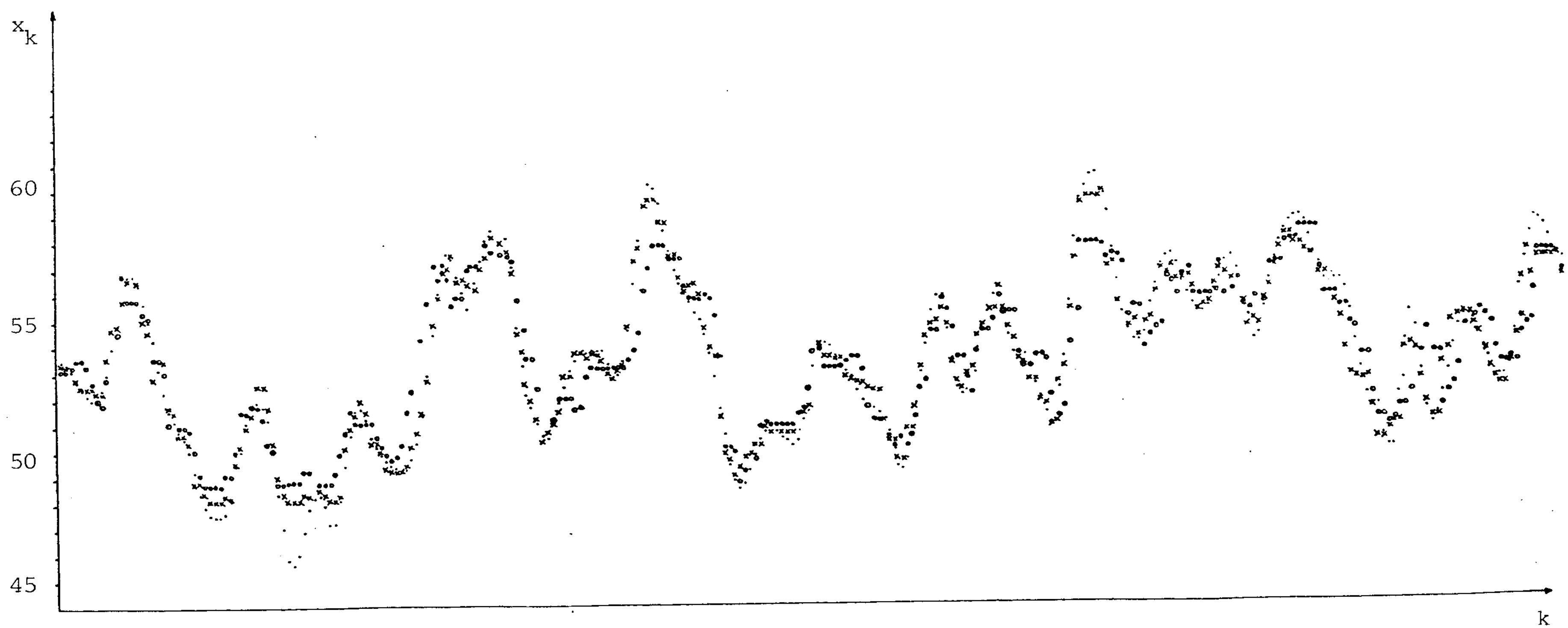

Fig.3 The fuzzy relational model for the Box-Jenkins data
o the model, $c_i=c_o=5$, × the model, $c_i=c_o=9$, • collected data

6. REFERENCES

1. J.C.Bezdek,Pattern Recognition With Fuzzy Objective Function Algorithms, Plenum Press,New York,1981.
2. G.E.P.Box,G.M.Jenkins,Time Series Analysis,Forecasting and Control. Holden Day,San Francisco,1970.
3. W.Pedrycz,Fuzzy Control and Fuzzy Systems,Dept.of Mathematics,Delft Univ. of Technology,82 14,1982.
4. H.Tanaka,S.Uejima,K.Asai,Fuzzy linear regression model,in Proc.of Int. Congress on Applied Systems Research and Cybernetics,G.E.Lasker ed., Vol.VI,2933-2938,Pergamon Press,New York,1981.
5. R.M.Tong,Synthesis of fuzzy models for industrial processes-some recent results,Int.J.General Systems,4,1978,143-162.
6. R.M.Tong,The evaluation of fuzzy models derived from experimental data, Fuzzy Sets and Systems,4,1980,1-12.
7. R.R.Yager,Fuzzy prediction based on regression models,Information Sciences, 26,1982,45-63.
8. D.Willaeys,Contribution a l'étude de la theorie des sous-ensembles flous en vue de son application a l'automatique,Ph.D.Thesis,Universitéde de Valenciennes et du Hainaut Cambresis,1980.
9. L.A.Zadeh,Toward a theory of fuzzy systems,in Aspects on Network and System Theory,R.E.Kalman,N.DeClaris eds.,Holt,Rinehart and Winston,New York,1971.
10. L.A.Zadeh,Fuzzy sets as a basis for a theory of possibility,Fuzzy Sets and Systems,1,1978,3-28.