SPECIAL FUZZY CARDINALITIES[§]

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We proposed in /1/ a new definition of fuzzy cardinality of finite fuzzy subsets, namely the Cd-cardinality. More precisely, let A denote a finite fuzzy subset of some universe U. Assume cardsuppA=n. Moreover, let $N:=\{0,1,2,\ldots\}$ and $I:=[0,1]$, where := denotes "equals by the definition". Then the Cd-cardinality of A is defined as a fuzzy natural number (fn-number) Cd_A, i.e. $Cd_A: N \rightarrow I$. Membership grades in Cd_A are expressed by the formula

$$Cd_A(k):=(0 \text{ if } P_k(A) \text{ is empty, else } \max\{dist(A,Y):\ Y \in P_k(A)\}),$$

where $P_k(A)$ denotes the family of k-element crisp subsets of the support of A and $dist(A,B):=\min_{x\in U}(A(x)\leftrightarrow B(x))$ with $a\leftrightarrow b:=1-|a-b|$ for $a,b\in I$ (Łukasiewicz equivalence operator). One can consider $Cd_A(k)$ to be degree to which A has <u>exactly</u> k elements. Properties of Cd_A were listed in /1/.

In this note we shall introduce, using Cd_A, fn-numbers defining degrees to which A has (resp.) less/greater than and at most/least k elements. Let $P_{\geqslant k}(A):=\{Y:\ Y\in P_j(k), j\geqslant k\}$ and $P_{>k}(A):=\{Y: Y\in P_j(k), j>k\}$. Then fn-numbers $Cd_A^{\geqslant}$, $Cd_A^{>}$, $Cd_A^{<}$, and $Cd_A^{\leqslant}$ are defined in the following way:

[§]This is abstract of a section of the paper "On fuzzy cardinalities and fuzzy binomial coefficient"(submitted to Fuzzy Sets and Systems Journal).

(i)

$$Cd_A^{\geqslant}(k) := \begin{cases} \max\{dist(A,Y):\ Y\in P_{\geqslant k}(A)\}, \\ 0 \text{ if } P_{\geqslant k}(A) \text{ is empty}, \\ 1 \text{ if } k=0, \end{cases}$$

(ii)

$$Cd_A^{>}(k) := \begin{cases} \max\{dist(A,Y):\ Y\in P_{>k}(A)\}, \\ 0 \text{ if } P_{>k}(A) \text{ is empty}, \end{cases}$$

(iii) $Cd_A^{<}(k) := 1 - Cd_A^{\geqslant}(k)$,

(iv) $Cd_A^{\leqslant}(k) := 1 - Cd_A^{>}(k)$.

Values $Cd_A^{\geqslant}(k)$, $Cd_A^{>}(k)$, $Cd_A^{<}(k)$ and $Cd_A^{\leqslant}(k)$, resp., will be called degrees to which A has (resp.) <u>at least</u>, <u>greater than</u>, <u>less than</u> and <u>at most</u> k elements.

Let us introduce, for fn-numbers F and G, the "vectorial" notations $F=(f_0,f_1,\dots,f_r)$ and $G=(g_0,g_1,\dots,g_{s-1},(g_s))$, where numbers f_i and g_j ($i=0,1,\dots,r$, $j=0,1,\dots,s$) denote,resp., membership grades of i in F (j in G) with the understanding that membership grades of $r+1,r+2,\dots$ ($s+1,s+2,\dots$) in F (in G) are equal to zero (to g_s). Thus $(f_0,f_1,\dots,f_s)=(f_0,f_1,\dots,f_s,(0))$. Moreover, let a_i denote the i-th element in descending sequence of the positive membership grades in A. Additionally, we put $a_0:=1$, $a_{n+1}:=0$, $p:=\min\{l:\ a_l+a_{l+1}\leqslant 1\}$, $d:=\max(a_p,1-a_p)$ and $d^c:=1-d$. If A differs from empty set, then we get:

(i) $Cd_A^{\geqslant}=(1,d,d,\dots,d,a_p,a_{p+1},\dots,a_n)$,

(ii) $Cd_A^{>}=(d,d,\dots,d,a_p,a_{p+1},\dots,a_n)$,

(iii) $Cd_A^{<}=(0,d^c,d^c,\dots,d^c,1-a_p,1-a_{p+1},\dots,1-a_n,(1))$,

(iv) $Cd_A^{\leqslant}=(d^c,d^c,\dots,d^c,1-a_p,1-a_{p+1},\dots,1-a_n,(1))$,

where the sequences consisting of numbers d (d^c, resp.) have always p-1 elements. For empty A we at once obtain

$$Cd^{\geqslant}_A=(1),\ Cd^{>}_A=(0),\ Cd^{<}_A=(0,(1)) \text{ and } Cd^{\leqslant}_A=((1)).$$

We noticed in /1/ that each membership grade $FGCount_A(k)$ in the fn-number $FGCount_A=(a_0,a_1,\ldots,a_n)$ (see /1/,/2/) defines degree to which A has <u>at least</u> rather than <u>exactly</u> k elements. It is easy to observe that $Cd^{\geqslant}_A=FGCount_A$ iff p=0,1. Moreover

(v) $Cd^{>}_A=(Cd^{\leqslant}_A)^c$ and $Cd^{<}_A=(Cd^{\geqslant}_A)^c$,

(vi) $Cd^{>}_A\subset Cd^{\geqslant}_A$ and $Cd^{<}_A\subset Cd^{\leqslant}_A$,

(vii) $Cd^{>}_A + Cd^{\leqslant}_A = Cd^{<}_A + Cd^{\geqslant}_A =((1))$,

(viii) $Cd^{>}_A.Cd^{<}_A=Cd^{>}_A.Cd^{\leqslant}_A=Cd^{<}_A.Cd^{\geqslant}_A=(0)$,

where $(F)^c$,+, . denote (resp.) complement of F, bounded-sum and bounded-product of two fuzzy subsets.

References

/1/ M.Wygralak, A new approach to the fuzzy cardinality of finite fuzzy subsets (the previous note).

/2/ L.A.Zadeh, Fuzzy probabilities and their role in decision analysis, in: Proc. IFAC Symp. on Th. and Appl. of Digital Control, New Dehli(January, 1982).