

A. APPLICATION OF FUZZY LOGIC AND APPROXIMATE REASONING

Jozef Šajda, Bratislava

In fuzzy logic formulas and the formulas of the classical propositional logic are known to be the same from the syntactical point of view. Differences would arise when one likes to interpret the formulas and define their truth functions. In this paper the truth function of fuzzy formulas is defined, it is going to be the one of Kandel and Lee.

DEFINITION OF THE TRUTH-FUNCTION FOR FUZZY FORMULAS

Let p be the truth-function, $p: \mathcal{F} \rightarrow [0,1]$, defined on the set \mathcal{F} of all fuzzy first-order formulas F with the values $p(F)$ in interval $[0,1]$ of real numbers. Values of the truth-function for compound formulas are defined as follows:

$$\begin{aligned} p(\neg F) &= 1 - p(F) \\ p(F \wedge G) &= \min\{p(F), p(G)\} \\ p(F \vee G) &= \max\{p(F), p(G)\} \\ p[\forall x] F &= \min\{p[F(x_1)], p[F(x_2)], \dots\} \\ p[\exists x] F &= \max\{p[F(x_1)], p[F(x_2)], \dots\} \end{aligned}$$

Where F and G are arbitrary fuzzy formulas with the truth-functions $p(F)$ and $p(G)$ defined before. From the definitions we can get according to the well-known equations

$$\begin{aligned} F \rightarrow G &\leftrightarrow \neg F \vee G \\ F \leftrightarrow G &\leftrightarrow (F \rightarrow G) \wedge (G \rightarrow F) \end{aligned}$$

It is possible to derive the truth-values

$$\begin{aligned} p(\neg \rightarrow) &= \max \{ p(\neg F), p(G) \} \\ p(\neg \leftrightarrow) &= \min \left\{ \max [p(\neg F), p(G)], \max [p(\neg G), p(F)] \right\} \end{aligned}$$

* Application and equivalence formulas.

The boundary properties of the functions min and max imply the possibility to extend the truth-function of conjunction and disjunction to a finite number of subformulas:

$$\begin{aligned} p(\neg F_1 \wedge \dots \wedge F_n) &= \min \{ p(F_1), p(F_2), \dots, p(F_n) \} \\ p(\neg F_1 \vee \dots \vee F_n) &= \max \{ p(F_1), p(F_2), \dots, p(F_n) \} \end{aligned}$$

* The derived truth-values of fuzzy formulas according to the inference rules

In addition to the inference rules which are implied by operational properties of conjunction, disjunction and negation we add more basic inference rules:

1. Rule of concretization expressed by the formula

$$(\forall_{x \in X}) F(x) \rightarrow F(a), \quad a \in X$$

2. Rule of modus ponens expressed by the scheme of form

$$\frac{\begin{array}{c} F(a) \\ (\forall x) [F(x) \rightarrow G(x)] \end{array}}{G(a)}$$

From the input truth-values, we define the output truth-value $\{G(a)\}$ in the case of concretization by

$$p[F(a)] = p[\forall x]F(x) = \min \{p[F(x_1)], p[F(x_2)], \dots\}, \quad x \in X$$

and calculate the output truth-value $p[G(a)]$ in the case of model a :

$$\begin{aligned} p[G(a)] &= p\{F(a) \wedge (\forall x)[F(x) \rightarrow G(x)]\} \\ &= \min \{p[F(a)], p[(\forall x)[F(x) \rightarrow G(x)]]\} \end{aligned}$$

Applications to a model of an organization

The multi-defined above are used to express a situation ^{given} which is modelled. In an organization structure attributes of the model according to personal properties of the employees are expressed by fuzzy formulas, the function of the model is defined by a set of basic principles expressed in the form of implications. Several nontrivial interesting issues about some structural properties of the model are obtained from the given ^{given} information using the inference rules ^{above}.